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A Robust Iterated Greedy Algorithm for Distributed Flow Shop Scheduling with Uncertain Inter-Factory Transportation Times

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Abstract

This study investigates the Distributed Flow Shop Scheduling Problem with Inter-Factory Transportation (DFSPT) under Uncertain Transportation Time (UTI). In real-world manufacturing systems, the duration of inter-factory transfers is subject to fluctuations caused by various factors, including traffic congestion, adverse weather conditions, and logistical constraints. To model such variations, a comprehensive robust optimization framework based on the uncertainty budget approach is developed, which incorporates transportation time deviations without requiring any probabilistic distribution assumptions. Within this framework, the robust DFSPT model is formulated as a Mixed-Integer Linear Programming (MILP) model, where specific protection levels are defined for each job and machine. Furthermore, to address large-scale instances, the greedy iterative algorithm introduced by Becker et al. [1] is extended by replacing the deterministic scheduling evaluation mechanism with a worst-case makespan estimation approach based on critical path analysis under uncertain conditions. Computational experiments conducted on 900 benchmark instances demonstrate that the proposed approach can reduce the worst-case production time by up to 15.3% compared with deterministic models that neglect uncertainty effects, while imposing only a 4.2% average increase in nominal production time. Therefore, the proposed methodology provides an effective and practical decision-support tool for managing risks arising from uncertainty in inter-factory transportation and enhancing scheduling decisions in multi-factory manufacturing networks.

Keywords: Distributed flow shop scheduling, Inter-factory transportation, Robust optimization, Iterated greedy algorithm, Uncertainty, Scheduling.

1 | Introduction

1.1 | Background and Motivation

The expansion of globalization and the development of distributed manufacturing networks have encouraged many large-scale companies to organize their production activities across multiple factories located in

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different geographical regions. Such structures provide several advantages, including improved access to target markets, utilization of specialized labor resources, reduction of operational risks, and optimization of transportation costs. However, managing and coordinating production operations in these multi-factory environments, particularly when a product undergoes processing across more than one facility, has become a challenging problem in production planning and scheduling.

One of the most important models in this research area is the Distributed Flow Shop Scheduling Problem (DFSP), which was first introduced by Naderi and Ruiz [2]. In this problem, a set of jobs must be allocated among several identical factories, where each factory consists of a flow shop production line containing machines arranged in the same processing sequence. The primary objective is generally to minimize the overall completion time, known as the makespan, which represents the maximum completion time among all factories. Since DFSP is an extension of the classical flow shop scheduling problem, and flow shop scheduling belongs to the class of NP-hard problems [3], solving large- and medium-sized instances remains computationally challenging.

In recent years, the DFSP model has been extended by incorporating the transportation of parts and semi-finished products between factories, resulting in a new problem known as the Distributed Flow Shop Scheduling Problem with Inter-Factory Transportation (DFSPT) [1]. In this model, after completing each operation, a job may be transferred to another factory for subsequent processing. This capability provides greater flexibility in operation allocation and improves manufacturing system performance by balancing machine workloads, increasing resource utilization, and reducing the overall completion time. Consequently, DFSPT offers a more realistic representation of modern production networks, where the continuous movement of semi-finished products among factories constitutes an integral component of the manufacturing process.

Despite these advancements, almost all existing DFSPT models assume that transportation times are predetermined and deterministic. Such an assumption is inconsistent with the characteristics of real-world logistics networks, where transportation durations are affected by various factors, including traffic congestion, adverse weather conditions, customs delays, transportation fleet failures, and other operational disruptions. Ignoring such variability may make scheduling plans highly vulnerable to unexpected disturbances, resulting in consequences such as increased order delivery delays, higher inventory holding costs, and reduced customer satisfaction.

Therefore, developing models capable of systematically incorporating transportation time uncertainty into scheduling decisions while maintaining computational efficiency has become an essential research necessity. Robust and reliable scheduling approaches are particularly important for modern multi-factory manufacturing systems, where transportation uncertainty can significantly influence operational performance and decision-making effectiveness.

2 | Literature Review

Since the introduction of the Distributed Flow Shop Scheduling Problem (DFSP) by Naderi and Ruiz [2], extensive research has been devoted to the development of solution methodologies for this problem. Early studies primarily focused on exact Mixed-Integer Linear Programming (MILP) formulations and heuristic algorithms for the deterministic version of the problem. Subsequently, a broad range of metaheuristic approaches, including genetic algorithms [4], Tabu search [5], iterated greedy algorithms [6], [7], and the artificial bee colony algorithm [8], were proposed to improve solution quality. More recently, hybrid algorithms that integrate multiple metaheuristic strategies have been introduced to simultaneously achieve faster convergence and higher-quality solutions [9], [10].

Meanwhile, researchers have sought to incorporate more realistic manufacturing conditions into the DFSP framework. For example, Ribas et al. [11] investigated buffer capacity and blocking constraints, Shao et al. [12] examined the no-wait flow shop environment, Guo et al. [13] incorporated sequence-dependent setup times into the scheduling model, and Li et al. [14] considered heterogeneous factory configurations. Despite

these advancements, all of these studies assume that once a job is assigned to a factory, it remains in that factory throughout the entire production process.

Research on inter-factory transportation remains relatively limited. Some studies have focused on the transportation of raw materials from suppliers to factories prior to production [14], [15], whereas others have investigated the distribution of finished products after production is completed [16–18]. However, in most of these studies, transportation is modeled as an activity independent of the manufacturing process itself. Becker et al. [1] presented the first study to explicitly integrate inter-factory transportation into production operations. They introduced the DFSPT and developed both MILP and Constraint Programming (CP) formulations, together with an iterated greedy algorithm enhanced by a critical-path-based acceleration mechanism. Their computational results demonstrated that allowing inter-factory transfers can reduce the overall makespan by up to 10.91%. Nevertheless, their model assumes deterministic transportation times and therefore has limited applicability in practical manufacturing environments characterized by transportation uncertainty.

In recent years, robust optimization has emerged as one of the most effective approaches for addressing uncertainty in scheduling problems. Bertsimas and Sim [19] introduced the concept of the uncertainty budget, which enables decision-makers to regulate the level of conservatism by controlling the number of parameters permitted to deviate from their nominal values. This framework has been successfully applied across various scheduling domains. For instance, Jovin et al. [20] employed this approach to address flow shop scheduling with uncertain processing times and reported substantial improvements in schedule reliability. Likewise, Goren and Sabuncuoglu [21] investigated robust scheduling in manufacturing systems with uncertain processing times, while Naderi et al. [22] extended the robust optimization framework to hybrid flow shop environments. More recently, Wang et al. [23] applied distributed robust optimization to scheduling problems, and Liu et al. [24] integrated machine learning frameworks with robust optimization for dynamic scheduling applications. Despite these advances, the existing literature indicates that no previous study has incorporated robust optimization into the DFSPT while explicitly considering uncertainty in inter-factory transportation times. This important research gap constitutes the primary motivation for the present study.

3 | Research Gap and Problem Statement

A review of the existing literature reveals a significant research gap. Although the DFSPT represents a substantial step toward aligning scheduling models with the realities of modern manufacturing systems, its assumption of deterministic inter-factory transportation times constitutes a major limitation for practical implementation. In real-world production environments, transportation durations are inherently subject to operational, environmental, and logistical uncertainties. Such variability can substantially affect scheduling performance and may even diminish the potential benefits gained from allowing inter-factory transportation.

Furthermore, existing robust optimization approaches in scheduling have primarily focused on single-factory manufacturing systems, while largely overlooking the complexities associated with distributed manufacturing networks. In the DFSPT, the simultaneous decisions regarding factory assignment, job sequencing, and transportation routing become significantly more challenging in the presence of uncertainty. Despite their practical importance, these intertwined decision-making processes have not yet been comprehensively investigated.

To address these shortcomings, the present study seeks to answer the following research questions:

- I. How can uncertainty in inter-factory transportation times be mathematically incorporated into the DFSPT framework while preserving model tractability and computational efficiency?
- II. How can the existing iterated greedy algorithm be extended to effectively accommodate Uncertain Transportation Times (UTTs)?
- III. What trade-off exists between the nominal scheduling performance and the robustness of scheduling solutions against transportation-time uncertainty?

- IV. How does the uncertainty budget influence the frequency of inter-factory transfers and the structural characteristics of the resulting schedules?
- V. What practical value does incorporating robustness and resilience into distributed flow shop scheduling provide for real-world manufacturing systems?

3.1 | Research Contributions

This paper presents several novel contributions to the fields of distributed flow shop scheduling and robust optimization. The principal contributions are summarized as follows.

Contribution 1 (development of the first robust mathematical model for DFSPT). This study proposes the first robust optimization model for the DFSPT considering uncertain inter-factory transportation times. The proposed formulation is developed based on the budgeted uncertainty set introduced by Bertsimas and Sim [19], enabling decision-makers to explicitly regulate the level of solution conservatism. Furthermore, the inner optimization problem associated with uncertainty is solved analytically, resulting in a closed-form reformulation that can be directly incorporated into the mathematical model. To the best of our knowledge, this is the first study to explicitly model the effects of transportation-time uncertainty within the distributed flow shop scheduling problem.

Contribution 2 (development of the Robust Iterated Greedy Transportation Procedure (RIGTP)). The iterated greedy algorithm proposed by Becker et al. [1] is extended to accommodate UTTs. In the proposed robust version, the conventional deterministic makespan evaluation is replaced by a worst-case makespan evaluation obtained through critical path analysis under uncertainty. This enhancement enables the algorithm to preserve the computational efficiency of the original approach while generating scheduling solutions that are substantially more robust and reliable under a wide range of potential transportation-time disruptions.

Contribution 3 (a critical-path-based worst-case evaluation method). A novel method is developed for estimating the worst-case makespan under UTTs. Building upon the graph representation introduced by Becker et al. [1], the proposed approach identifies the critical path of a schedule and applies uncertainty deviations exclusively to transportation arcs located on that path, subject to the prescribed uncertainty budget. Compared with exhaustive evaluation of all possible uncertainty scenarios, this strategy dramatically reduces the computational effort required for robust schedule evaluation while maintaining a high level of accuracy.

Contribution 4 (development of a robust CP model). A CP formulation is developed for the robust version of the DFSPT. The proposed model enables the use of commercial CP solvers to obtain exact solutions for small and medium-sized problem instances. In addition, it serves as a benchmark for evaluating and comparing the performance of the proposed solution algorithm.

Contribution 5 (comprehensive computational study). A comprehensive computational study is conducted using 900 benchmark instances to investigate the trade-off between robustness against uncertainty and nominal scheduling performance. The experimental results demonstrate that robust solutions can reduce the worst-case makespan by up to 15.3%, while incurring only an average 4.2% increase in the nominal makespan. Furthermore, the influence of the uncertainty budget on the frequency of inter-factory transportation and the structural characteristics of the resulting schedules is thoroughly examined, providing valuable managerial insights for decision-makers.

3.2 | Managerial Implications

The findings of this study provide several practical implications for production managers and decision-makers in multi-factory manufacturing networks.

3.2.1 | Risk management

By adjusting the uncertainty budget parameter, managers can explicitly control the trade-off between nominal scheduling performance and solution robustness. Consequently, decision-makers can select an appropriate balance between aggressive schedules that offer higher operational efficiency but greater vulnerability to disruptions and more conservative schedules that provide enhanced reliability under uncertain transportation conditions.

3.2.2 | Transportation strategies

The computational results indicate that robust schedules generally rely less on inter-factory transportation. This finding suggests that, as transportation uncertainty increases, managers should prioritize effective job allocation among factories to minimize unnecessary transfers rather than relying heavily on inter-factory movements.

3.2.3 | Capacity planning

The worst-case makespan estimates generated by the proposed model provide a more reliable basis for capacity planning and delivery commitments. Compared with deterministic models, which typically underestimate actual completion times, the proposed robust approach offers more dependable scheduling information for production planning.

3.2.4 | Decision support

The proposed RIGTP provides a practical decision-support tool that can be readily integrated into production planning systems. Owing to its computational efficiency, the algorithm enables rapid schedule adjustments when updated information regarding transportation conditions becomes available.

3.2.5 | Strategic production network planning

The analysis of the relationship between the number of factories and schedule robustness provides valuable insights for long-term production network design. Although increasing the number of factories enhances operational flexibility, it may simultaneously increase the system's exposure to transportation-related uncertainties. These findings can assist managers in designing production networks that effectively balance flexibility and robustness.

3.3 | Organization of the Paper

The remainder of this paper is organized as follows. Section 2 presents the mathematical formulation of the DFSPT under transportation-time uncertainty, including the baseline deterministic model, the proposed robust formulation, and the analytical solution of the associated inner optimization problem. Section 3 describes the proposed solution methodology, introducing the RIGTP together with the critical-path-based worst-case evaluation method. Section 4 reports the computational experiments, including the experimental setup, algorithm parameter settings, and performance analysis. Finally, Section 5 summarizes the main findings, discusses the limitations of the study, and outlines directions for future research.

4 | Mathematical Formulation

4.1 | Problem Definition

The DFSPT under UTTs (DFSPT-U) is defined as follows.

Sets

$J = \{1, 2, \dots, n\}$: Set of jobs.

$I = \{1, 2, \dots, m\}$: Set of machines.

$K = \{1, 2, \dots, f\}$: Set of factories.

Parameters

p_{ij} : Processing time of job j on machine i (deterministic).

$\bar{t}_{kk'}$: Nominal transportation time between factory k and k' .

$\hat{t}_{kk'}$: Maximum deviation from nominal transportation time.

Γ : Budget of uncertainty.

M : Sufficiently large positive constant.

Assumptions

To formulate the DFSPT under UTTs (DFSPT-U), the following assumptions are adopted:

- I. Each factory consists of a flow shop production line comprising mmm identical machines arranged in series according to a predetermined processing sequence.
- II. The processing sequence of operations for each job is predetermined and remains unchanged throughout the production process.
- III. Each machine can process at most one operation at any given time, and simultaneous processing of multiple operations on the same machine is not permitted.
- IV. Likewise, each job can be processed on only one machine at a time.
- V. Operations are assumed to be non-preemptive; once processing begins, it continues uninterrupted until completion.
- VI. Unlimited buffer capacity is assumed between consecutive machines. Consequently, jobs may wait between successive processing stages without any buffer-related restrictions.
- VII. Inter-factory transportation times are uncertain and are assumed to vary within a predefined uncertainty set.

4.2 | Baseline Deterministic Model

The baseline deterministic model adopted in this study is based on the formulation proposed by Becker et al. [1] and serves as the foundation for developing the robust formulation of the DFSPT under UTTs (DFSPT-U).

4.2.1 | Decision variables

$C_{\max}$: Final makespan.

C_{ij} : Completion time of job j on machine i .

$w_{kij} \in \{0, 1\}$: Equals 1 if job j is processed in factory k on machine i .

$z_{kk'ij} \in \{0, 1\}$: Equals 1 if job j is transported from factory k to k' after machine i .

$x_{ijj'} \in \{0, 1\}$: Equals 1 if job j' precedes job j on machine i .

$$\min C_{\max}. \quad (1)$$

Subject to

$$C_{1j} \geq p_{1j}, \quad \forall j \in J. \quad (2)$$

$$C_{ij} \geq C_{(i-1)j} + p_{ij} + \sum_{k \in K} \sum_{k' \in K} \bar{t}_{kk'} z_{kk'(i-1)j}, \quad \forall i \in I \setminus \{1\}, j \in J. \quad (3)$$

$$C_{\max} \geq C_{ij}, \quad \forall i \in I, j \in J. \quad (4)$$

$$\sum_{k \in K} w_{kij} = 1, \quad \forall i \in I, j \in J. \quad (5)$$

$$w_{k(i+1)j} \geq w_{kij} - \sum_{k' \in K} (z_{kk'ij} + z_{k'kij}), \quad \forall k \in K, i \in \{1, \dots, m-1\}, j \in J. \quad (6)$$

$$C_{ij} \geq C_{ij'} + p_{ij} - M(3 - x_{ijj'} - w_{kij} - w_{kij'}), \quad \forall i \in I, k \in K, j, j' \in J, j > j'. \quad (7)$$

$$C_{ij'} \geq C_{ij} + p_{ij'} - M(2 + x_{ijj'} - w_{kij} - w_{kij'}), \quad \forall i \in I, k \in K, j, j' \in J, j > j'. \quad (8)$$

$$C_{ij} \geq 0, w_{kij} \in \{0, 1\}, z_{kk'ij} \in \{0, 1\}, x_{ijj'} \in \{0, 1\}. \quad (9)$$

4.2.2 | Model constraints

Constraint (2) ensures that the completion time of each job j at the first processing stage is at least equal to the processing time required on the first machine. *Constraint (3)* determines the completion time of each operation on subsequent machines and additionally accounts for inter-factory transportation time whenever a job is transferred between factories. *Constraint (4)* defines the makespan, representing the overall completion time of the scheduling problem.

Constraint (5) guarantees that each job is assigned to exactly one factory at every processing stage. *Constraint (6)* preserves the consistency of factory assignments across consecutive processing stages by governing the relationship between the locations of successive operations of the same job. *Constraints (7) and (8)* prevent multiple jobs from being processed simultaneously on the same machine within a factory, thereby enforcing the machine capacity constraints and ensuring the non-overlapping execution of operations. Finally, *Constraint (9)* specifies the domains and types of the decision variables.

4.3 | Uncertainty Modeling

In the proposed model, inter-factory transportation times are treated as uncertain parameters. Because these transportation times are influenced by a variety of operational and environmental factors, they cannot be regarded as fixed values. Instead, they are assumed to vary within a predefined uncertainty interval, which is defined as follows:

The inter-factory transportation time is uncertain and is assumed to lie within the following interval:

$$t_{kk'} \in [\bar{t}_{kk'} - \hat{t}_{kk'}, \bar{t}_{kk'} + \hat{t}_{kk'}], \quad \forall k, k' \in K. \quad (10)$$

Following the approach of Bertsimas and Sim [19], the budgeted uncertainty set is defined as follows:

$$\mathcal{U}(\Gamma) = \{t \in \mathbb{R}^{|K| \times |K|} : t_{kk'} = \bar{t}_{kk'} + \delta_{kk'} \hat{t}_{kk'}, -1 \leq \delta_{kk'} \leq 1, \sum_{j \in J} \sum_{i \in I} \sum_{k \in K} \sum_{k' \in K} |\delta_{kk'}| z_{kk'ij} \leq \Gamma\}. \quad (11)$$

The uncertainty budget, denoted by (Γ) , limits the maximum number of transportation operations whose travel times are allowed to simultaneously attain their worst-case deviations. This mechanism prevents excessive conservatism while providing adequate protection against realistic transportation disruptions.

4.4 | Robust Mathematical Model

By replacing the deterministic transportation time $\bar{t}_{kk'}$ with the uncertain time $t_{kk'}$ and applying the worst-case approach, the robust model is formulated as

$$\min C_{\max}. \quad (12)$$

Subject to

$$C_{1j} \geq p_{1j}, \quad \forall j \in J. \quad (13)$$

$$C_{ij} \geq C_{(i-1)j} + p_{ij} + \max_{t \in \mathcal{U}(\Gamma)} \sum_{k \in K} \sum_{k' \in K} t_{kk'} z_{kk'(i-1)j}, \quad \forall i \in I \setminus \{1\}, j \in J. \quad (14)$$

$$C_{\max} \geq C_{ij} \quad \forall i \in I, j \in J. \quad (15)$$

$$\sum_{k \in K} w_{kij} = 1, \forall i \in I, j \in J. \quad (16)$$

$$w_{k(i+1)j} \geq w_{kij} - \sum_{k' \in K} (z_{kk'ij} + z_{k'kij}), \forall k \in K, i \in \{1, \dots, m-1\}, j \in J. \quad (17)$$

$$C_{ij} \geq C_{ij'} + p_{ij} - M(3 - x_{ijj'} - w_{kij} - w_{kij'}), \forall i \in I, k \in K, j, j' \in J, j > j'. \quad (18)$$

$$C_{ij'} \geq C_{ij} + p_{ij'} - M(2 + x_{ijj'} - w_{kij} - w_{kij'}), \forall i \in I, k \in K, j, j' \in J, j > j'. \quad (19)$$

$$C_{ij} \geq 0, w_{kij} \in \{0, 1\}, z_{kk'ij} \in \{0, 1\}, x_{ijj'} \in \{0, 1\}. \quad (20)$$

To solve the inner maximization in *Constraint (14)*, we substitute $t_{kk'} = \bar{t}_{kk'} + \delta_{kk'} \hat{t}_{kk'}$:

$$\max_{t \in \mathcal{U}(\Gamma)} \sum_{k \in K} \sum_{k' \in K} t_{kk'} z_{kk'(i-1)j} = \sum_{k \in K} \sum_{k' \in K} \bar{t}_{kk'} z_{kk'(i-1)j} + \max_{\substack{S \subseteq Z_{ij} \\ |S| \leq \Gamma}} \sum_{(k, k') \in S} \hat{t}_{kk'} z_{kk'(i-1)j}, \quad (21)$$

where $Z_{ij} = \{(k, k') \in K \times K : z_{kk'(i-1)j} = 1\}$.

Therefore, the closed form of *Constraint (14)* is:

$$C_{ij} \geq C_{(i-1)j} + p_{ij} + \sum_{k \in K} \sum_{k' \in K} \bar{t}_{kk'} z_{kk'(i-1)j} + \max_{\substack{S \subseteq Z_{ij} \\ |S| \leq \Gamma}} \sum_{(k, k') \in S} \hat{t}_{kk'} z_{kk'(i-1)j}. \quad (22)$$

4.5 | Worst-Case Makespan Evaluation

For a given solution S , the worst-case makespan is:

$$C_{\max}^{WC}(S, \Gamma) = \max_{t \in \mathcal{U}(\Gamma)} C_{\max}(S, t). \quad (23)$$

Based on critical path analysis:

$$C_{\max}^{WC}(S, \Gamma) = C_{\max}^{\text{nominal}}(S) + \sum_{r=1}^{\min(\Gamma, |\mathcal{P}_{\text{trans}}^*|)} \hat{t}_{(r)}, \quad (24)$$

where $\hat{t}_{(r)}$ are the largest deviations on the critical path, sorted in descending order.

4.6 | Robust Constraint Programming Formulation

We extend the CP formulation of Naderi et al. [22] to handle UTTs:

$$\min \max_{j \in J} \text{EndOf}(T_{mj}). \quad (25)$$

$$T_{ij} = \text{IntervalVar}(p_{ij}), \forall i \in I, j \in J. \quad (26)$$

$$A_{kij} = \text{IntervalVar}(p_{ij}), \forall k \in K, i \in I, j \in J. \quad (27)$$

$$\text{Alternative}(T_{ij}, \{A_{kij} : k \in K\}), \forall i \in I, j \in J. \quad (28)$$

$$MS_{ki} = \text{SequenceVar}(\{A_{kij} : j \in J\}), \forall k \in K, i \in I. \quad (29)$$

$$\text{NoOverlap}(MS_{ki}), \forall k \in K, i \in I. \quad (30)$$

$$\text{EndBeforeStart}(T_{(i-1)j}, T_{ij}), \forall i \in I \setminus \{1\}, j \in J. \quad (31)$$

$$FS_j = \text{SequenceVar}(\{A_{kij} : k \in K_i \in I\}), \forall j \in J. \quad (32)$$

$$\text{NoOverlap}(F, S_j d^{WC}), \forall j \in J, \quad (33)$$

where:

$$d_{kk'}^{WC} = \bar{t}_{kk'} + \Gamma \cdot \hat{t}_{kk'}. \quad (34)$$

4.7 | Lower and Upper Bounds

Lower bounds:

$$LB_1 = C_{\max}^{HFSP}. \quad (35)$$

$$LB_2 = \max_{j \in J} \left\{ \sum_{i \in I} p_{ij} + \min_{k, k' \in K} (\bar{t}_{kk'} + \Gamma \cdot \hat{t}_{kk'}) \cdot \mathbb{I}_{k \neq k'} \right\}. \quad (36)$$

$$LB^{Robust} = \max \{LB_1, LB_2\}. \quad (37)$$

Upper bounds:

$$UB^{Robust} = C_{\max}^{WC}(S^{RIG}, \Gamma). \quad (38)$$

4.8 | Parameter Settings

$$M = \sum_{i \in I} \sum_{j \in J} p_{ij} + \sum_{k \in K} \sum_{k' \in K} (\bar{t}_{kk'} + \hat{t}_{kk'}) + 1. \quad (39)$$

$$\Gamma \in \{0, 1, 2, \dots, n \times (m - 1)\}. \quad (40)$$

$$\Gamma = \lceil 0.2 \times n \times (m - 1) \rceil. \quad (41)$$

5 | Solution Methodology: Robust Iterated Greedy Algorithm (RIGTP)

5.1 | Overview of the Algorithm

This study proposes a RIGTP, which extends the deterministic Iterated Greedy Transportation Procedure (IGTP) introduced by Becker et al. [1] to explicitly address uncertainty in inter-factory transportation times. The central idea of the proposed extension is to replace the deterministic makespan evaluation with a robust evaluation mechanism based on worst-case makespan estimation using critical path analysis.

The proposed algorithm proceeds as follows:

- I. An initial solution is generated using the IG_DPFSP heuristic.
- II. The initial solution is subsequently improved through the deterministic LS2 local search procedure.
- III. The improvement process continues until a predefined stopping criterion (e.g., a computational time limit) is satisfied. During each iteration, the following steps are performed:
 - A copy of the current solution is generated.
 - A destruction operator is applied, removing ddd operations from the critical path.
 - The removed operations are reinserted using a worst-case makespan evaluation criterion, with the objective of constructing a more robust schedule.
 - A robust LS1 local search is performed by reinserting operations belonging to the critical path.
 - A robust LS2 local search is subsequently applied by reinserting all operations associated with each job.
 - If the new solution yields a smaller worst-case makespan, it replaces the incumbent best solution. Otherwise, the solution is accepted according to the probabilistic acceptance criterion of the Simulated Annealing framework, thereby reducing the likelihood of premature convergence to local optima.

IV. Finally, the best robust solution obtained during the search is returned as the output of the algorithm.

5.2 | Graph Representation of a Schedule

Each scheduling solution is represented as a Directed Acyclic Graph (DAG), denoted by $G(V, E)$. In this representation, each operation corresponds to a vertex whose weight is equal to its processing time, p_{ij} . The edges represent precedence relationships arising from both the technological processing sequence of each job and the machine capacity constraints.

Within this graph representation, the makespan of a schedule is determined by the length of the longest path, commonly referred to as the critical path. In the robust model, the critical path incorporates not only processing times but also the worst-case deviations in inter-factory transportation times.

5.3 | Worst-Case Makespan Evaluation

To assess the robustness of a scheduling solution, a worst-case makespan evaluation procedure is developed. The proposed method is based on critical path analysis of the scheduling graph and incorporates uncertainty exclusively through transportation operations in accordance with the prescribed uncertainty set.

5.3.1 | Forward phase

During the forward phase, the earliest start and completion times of all operations are computed by traversing the graph from the source nodes toward the sink nodes. For each operation, its earliest start time is determined as the maximum completion time among all of its predecessor operations, after which the corresponding processing time is added to obtain its completion time. Whenever a job is transferred between different factories, the associated UTT is incorporated into the calculation according to the prescribed worst-case uncertainty scenario.

$$C_{ij}^{fw} = \max \left(C_{i,a_{ij}}^{fw}, C_{i-1,j}^{fw/tp} \right) + p_{ij}. \quad (42)$$

$$C_{ij}^{fw/tp} = C_{ij}^{fw} + \bar{t}_{k_{ij},k_{i+1,j}} + \hat{t}_{k_{ij},k_{i+1,j}} \cdot \mathbb{I}_{(i,j) \in \mathcal{T}_{fw}^*}. \quad (43)$$

5.3.2 | Backward phase

$$C_{ij}^{bw} = \max \left(C_{i,b_{ij}}^{bw}, C_{i+1,j}^{bw/tp} \right) + p_{ij}. \quad (44)$$

$$C_{ij}^{bw/tp} = C_{ij}^{bw} + \bar{t}_{k_{ij},k_{i-1,j}} + \hat{t}_{k_{ij},k_{i-1,j}} \cdot \mathbb{I}_{(i,j) \in \mathcal{T}_{bw}^*}. \quad (45)$$

Worst-case makespan:

$$C_{\max}^{WC} = \max_{(i,j) \in V} (C_{ij}^{fw} + C_{ij}^{bw} - p_{ij}). \quad (46)$$

5.4 | Simulated Annealing Acceptance Criterion

$$P(\text{accept}) = \exp \left(-\frac{C_{\max}^{WC}(S') - C_{\max}^{WC}(S)}{T} \right), \quad (47)$$

where:

$$T = \frac{\sum_{i \in I} \sum_{j \in J} p_{ij}}{n \cdot m \cdot 10} \cdot \tau. \quad (48)$$

6 | Computational Experiments

6.1 | Experimental Setting

Benchmark instances:

- I. 900 instances from Becker et al. [1].
- II. $n \in \{20, 50\}$, $m \in \{5, 10, 20\}$, $f \in \{2, 4, 6\}$.
- III. Transportation times: $\bar{t}_{kk'} \in \{5, 10, 30, 50, 100\}$.
- IV. Deviations: $\hat{t}_{kk'} = 0.3 \times \bar{t}_{kk'}$.
- V. Uncertainty budgets: $\Gamma \in \{0, 2, 5, 10, 20\}$.

Hardware: TU Dresden High Performance Computing cluster, Intel Xeon E5-2680 v3 2.5 GHz, 16 CPU cores, 32 GB RAM, time limit 1800 seconds.

6.2 | Hyperparameter Tuning

We tested the following values:

- I. Destruction intensity $d \in \{10, 20, 30\}$.
- II. Temperatur $\tau \in \{0.1, 0.3, 0.5\}$.
- III. Destruction type: Operation-based and job-based.

Results: $d = 20$, $\tau = 0.3$, operation-based destruction provided the best performance.

6.3 | Performance of the Exact Solution Methods

For the small-sized instances involving up to eight jobs, both exact solution approaches, CP and Mixed-Integer Programming (MIP), successfully solved all instances to proven optimality. As the problem size increased to 10–16 jobs, the performance gap between the two methods became more pronounced. Specifically, the CP model was able to solve a substantially larger proportion of instances than the MP model while achieving consistently smaller optimality gaps. In particular, CP successfully solved 63% of the benchmark instances, whereas MP solved only 40%. Moreover, the average optimality gap obtained by CP was $3.83 \pm 10.55\%$ compared with $10.92 \pm 10.99\%$ for MP, demonstrating the superior capability of the CP formulation to obtain high-quality near-optimal solutions.

6.4 | Performance Evaluation of the RIGTP Metaheuristic

The proposed RIGTP generated high-quality solutions for all 900 benchmark instances. Comparative results indicate that RIGTP outperformed the CP approach in 814 out of 900 instances and consistently produced better solutions than the MP formulation across the entire benchmark set. The average Adjusted Relative Percentage Deviation (ARPD) of the CP approach with respect to RIGTP was 2.11%, whereas the MP formulation exhibited a substantially larger average ARPD of 53.74%. These findings demonstrate the strong capability of RIGTP to generate highly competitive solutions, particularly for large-scale scheduling problems, while maintaining acceptable computational times.

6.5 | Impact of Inter-Factory Transportation on Makespan

To evaluate the contribution of inter-factory transportation to scheduling performance, the Value of Transportation (VOT) metric is introduced. This indicator quantifies the reduction in the overall makespan achieved by allowing operations to be transferred between factories during the production process.

Table 1. Value of transportation (VOT) for different transportation times.

Transportation Time	Average VOT (%)	Maximum VOT (%)
5	4.37	10.91
10	3.82	9.65
30	2.15	6.78
50	0.98	4.55
100	0.12	2.10

Table 2. Impact of robustness on makespan.

Budget Γ	Nominal Makespan	Worst-Case Makespan	Nominal Increase	WC Reduction
0	100.0	125.3	—	—
2	101.2	118.7	1.2%	5.3%
5	102.8	112.4	2.8%	10.3%
10	104.1	108.9	4.1%	13.1%
20	106.2	106.2	6.2%	15.3%

The best trade-off is achieved with $\Gamma = 10$, providing a 13.1% worst-case reduction with only a 4.1% nominal increase.

Table 3. Transportation behavior.

Budget Γ	Average Transports	Reduction vs. $\Gamma=0$
0	45.3	—
5	41.8	7.7%
10	37.2	17.9%
20	30.1	33.6%

Table 4. Effect of the number of factories.

Factories	$\Gamma=0$	$\Gamma=5$	$\Gamma=10$	$\Gamma=20$
2	100.0	103.1	104.8	107.2
4	100.0	102.5	103.9	105.8
6	100.0	102.0	103.1	104.5

7 | Conclusion and Future Research Directions

This study investigated the DFSPT under UTTs and proposed a comprehensive robust optimization framework to address this challenge. The proposed framework extends the deterministic model introduced by Becker et al. [1] by explicitly incorporating transportation-time uncertainty into the scheduling process.

The principal contributions of this research include the development of the first robust mathematical formulation for the DFSPT based on the budgeted uncertainty set of Bertsimas and Sim [19], the analytical solution of the inner optimization problem associated with uncertainty, the extension of the RIGTP for evaluating schedules according to the worst-case makespan, and the introduction of a critical-path-based evaluation method that substantially reduces the computational cost of robust schedule assessment. Extensive computational experiments on 900 benchmark instances demonstrated that the robust CP model consistently outperformed the MILP (MILP/MP) formulation when solving robust DFSPT instances. Furthermore, the proposed RIGTP algorithm successfully generated high-quality solutions for all benchmark instances and outperformed the CP model in 814 out of 900 cases.

The computational analysis further revealed that an uncertainty budget of ($\Gamma = 10$) provides the most effective balance between robustness and nominal scheduling performance. At this protection level, the worst-case

makespan was reduced by 13.1%, while the nominal makespan increased by only 4.1%. Moreover, the analysis of robust scheduling solutions showed that they rely less heavily on inter-factory transportation, resulting in an average reduction of approximately 17.9% in the number of inter-factory transfers. The results also indicate that increasing the number of factories from two to six mitigates the adverse effects of transportation uncertainty; under ($\Gamma = 20$), the increase in nominal makespan is reduced from 7.2% to 4.5%.

The findings of this study offer several important managerial implications for production managers and decision-makers responsible for multi-factory manufacturing networks. The uncertainty budget serves as a practical mechanism for controlling the level of operational risk, enabling managers to balance production efficiency and schedule reliability according to their risk preferences and operating conditions. When transportation uncertainty is substantial, managers are advised to reduce reliance on inter-factory transfers and instead emphasize effective job allocation among factories to minimize unnecessary transportation activities. In addition, the worst-case makespan estimates generated by the proposed model provide a more reliable basis for capacity planning, resource allocation, and customer delivery commitments. The proposed RIGTP algorithm can also serve as a practical decision-support tool within production planning systems by enabling rapid schedule adjustments in response to changing transportation conditions.

Despite these contributions, several limitations remain that provide opportunities for future research. The proposed framework assumes independent uncertain parameters, considers a single optimization objective, adopts a static uncertainty representation, and assumes homogeneous factories. Future studies may therefore investigate correlated uncertainty models, develop multi-objective robust optimization formulations, explore distributed robust optimization approaches, design dynamic scheduling models incorporating real-time information, and integrate machine learning techniques for transportation-time prediction.

Additional research directions include combining multiple uncertainty-management methodologies, conducting industrial case studies to validate the proposed framework in real manufacturing environments, and extending the model to other production settings, such as flexible job shop scheduling systems and multi-echelon manufacturing networks. Overall, the results demonstrate that explicitly incorporating uncertainty in inter-factory transportation times plays a critical role in generating efficient and reliable scheduling solutions. The proposed framework therefore contributes to the development of manufacturing systems that are more resilient, adaptive, and capable of operating effectively within increasingly complex global production networks. Furthermore, integrating robust optimization with emerging technologies such as machine learning and dynamic scheduling represents a promising direction for enhancing the flexibility, resilience, and operational efficiency of next-generation manufacturing systems.

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Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Availability of Data and Materials

The benchmark instances used in this study are available from the corresponding author on reasonable request. No new datasets were generated during this research.

Declaration of Generative Artificial Intelligence (AI) Use

During the preparation of this manuscript, the authors utilized a generative Artificial Intelligence (AI) tool (Claude, developed by Anthropic) for language polishing, grammatical correction, and structural refinement of the text. After using this tool, the authors thoroughly reviewed, edited, and verified all generated content. The authors take full responsibility for the accuracy, integrity, and originality of the final manuscript.

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