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A Solution of Multi-Level Multi-Objective Optimization Models for Solid Transportation Problems under Fuzzy Environments

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Abstract

This study presents a comprehensive multi-level, multi-objective mathematical model to address the Solid Transportation Problem (STP) under fuzzy environments. Real-world transportation systems often face significant complexity due to uncertain parameters such as fluctuating demand, varying traffic conditions, and limited conveyance capacity. To effectively manage these uncertainties, we propose a mathematical framework that incorporates hierarchical objectives and uncertain parameters using fuzzy set theory. The model is structured across three hierarchical levels, each representing a specific goal within the transportation system: 1) minimization of transportation costs, 2) minimization of total transportation time, and 3) minimization of environmental impact costs. To manage uncertainty within model parameters, we apply two robust techniques—expected constraint programming and chance constraint programming—which transform the fuzzy model into its deterministic equivalent, allowing for more practical implementation and decision-making. The proposed model is validated through a hierarchical multi-level, multi-objective numerical illustration. This example demonstrates the effectiveness and applicability of our approach in real-world scenarios. It highlights how the model supports decision-makers in navigating trade-offs among conflicting objectives while maintaining feasibility under uncertain conditions. The results underscore the model's potential to enhance efficiency, sustainability, and resilience in solid transportation operations.

Keywords: Optimization, Solid transportation problem, Fuzzy environment, Multi-level programming, Uncertainty.

1 | Introduction

Effective transportation decision-making plays a crucial role in maximizing the profitability of any firm or organization [1–3]. In today's highly competitive corporate landscape, organizations strive to develop transportation strategies that are both efficient and cost-effective [4]. Choosing the right transportation

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channels can significantly reduce expenses and delivery times, ensuring that products reach customers swiftly and at a lower cost [4]. It enhances operational efficiency and boosts the organization's overall profitability and competitive edge [5]. This study explores an advanced version of the traditional TP known as the Solid Transportation Problem (STP). In a traditional Transportation Problem (TP), the goal is to identify the most cost-effective routes for transporting goods while meeting specific demand and supply constraints. However, in real-world scenarios, multiple modes of transportation are often available and used to ship goods from sources to destinations. These diverse transportation options, called conveyances, add complexity to the decision-making process and are a crucial focus of this study [5], [6].

It highlights the importance of studying the STP, which incorporates three critical constraints: source, destination, and conveyance capacity. STP was first introduced by Schell [7] as a generalization of the traditional TP. Subsequently, Haley [8] developed the first procedure for solving STP. Later, Liu [9] presented a novel approach to finding a locally optimal basic feasible solution for an STP with an indefinite quadratic objective function, further advancing the field. *Fig. 1* presents a diagrammatic representation of an STP network, illustrating the flow of goods from multiple sources to various destinations through intermediate transshipment points. The diagram highlights the transportation system's interconnections, capacities, and cost considerations.

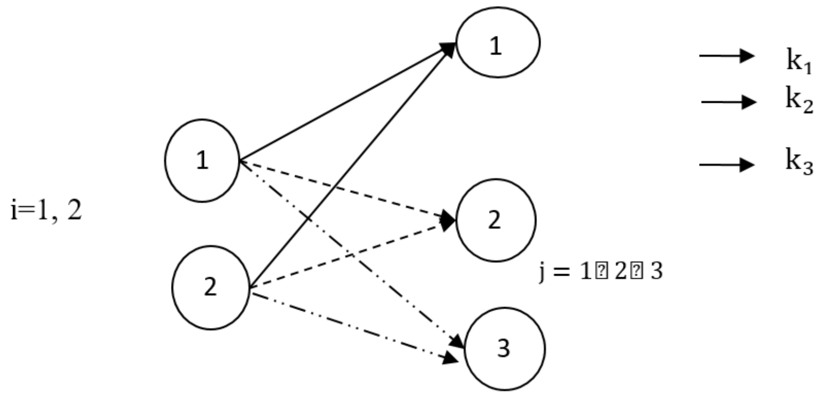

Fig. 1. An STP farmwork.

In real-world transportation firms, decision-making often follows a hierarchical structure among different levels of decision-makers. However, most existing studies on the STP using uncertain programming approaches have overlooked this hierarchy when formulating their models. In this study, we bridge this gap by presenting an STP model that closely aligns with the operational dynamics of real-life transportation firms. This study approaches the STP as a multi-level programming problem. In such problems, decision-making follows a hierarchical structure, where the Decision Maker (DM) at the top level solves their objective first. Subsequently, the DM at the next level addresses their objectives based on the information provided by the higher level. This process continues down the hierarchy, with each DM seeking to minimize or maximize their objectives, which are often interdependent on the variables controlled at other levels. This hierarchical framework is particularly suited for modeling STP, as it typically involves multiple DMs operating at different levels. Additionally, the hierarchy of objective functions reflects the priorities or preferences of the DMs regarding their respective goals, as highlighted by Peng et al. [10].

In real-world scenarios, variables in TP—such as transportation cost, time, supply, demand, and conveyance capacity—are often uncertain and not fixed. This underscores the importance of studying TP under uncertainty. To address this, the concept of uncertainty programming introduced by Liu & Liu [11] is employed to model the uncertainties in the proposed multi-level STP. Numerous researchers have previously explored transportation planning decision problems in uncertain environments. Varathan and Edward J [12] proposed an uncertain programming model for STP using expected-constrained programming. Ali et al. [1] tackled a multi-objective fixed-charge TP with uncertainty and solved this model using the Fermatean fuzzy programming approach. Joshi et al. [13] studied TP with random supply levels and uncertain cost parameters, addressing the challenges posed by stochastic and ambiguous factors. Kundu et al. [14] developed an STP model for product blending, incorporating parameter tables. Mondal et al. [5] analyzed a supply chain model with variable demand under a three-level trade credit policy.

Additionally, Azimian & Aouni [15] introduced an uncertain programming model for multi-item STP, further enriching the literature on TP in uncertain settings. The remaining work is organized as follows: Section 2 formulates the uncertain multi-level multi-objective STP model, while Section 3 provides its equivalent crisp representation. Section 4 introduces a fuzzy logic-based solution methodology to address the multi-level, multi-objective problem. Section 5 illustrates the approach with a numerical example of a three-level STP. Section 6 discussed the result and discussion of the proposed problem. Finally, Section 7 discusses the conclusion of the proposed study.

2 | Uncertain Multi-Level Multi-Objective Solid Transportation Problem

A multi-level programming problem deals with situations involving more than one objective to be optimized in a hierarchical order. The objectives are assigned levels according to the order of their preferences. An objective with the highest preference order is placed at level 1st and is known as the leader. STP can be studied as a multi-level optimization problem, as in real cases, it may involve more than one objective, such as the minimization of some crucial parameters, such as the total transportation cost, the total number of items defective during transportation, total shipment time, the number of goods to be delivered and so on [3], [16]. These objectives can be arranged in hierarchical order according to the DM's preferences. In the balance STP, all three item properties (i.e., the sum of supplies, demand, and conveyance capacities) should be equal. However, generally, it seldom happens, and in most cases, the balance condition does not hold. Here, we have taken the case of an unbalanced STP. Let us assume a decision-making system with t DMs and t objectives to be minimized hierarchically. The following nomenclatures presented in *Table 1* are used throughout the study to model the uncertain multi-level multi-objective STP.

Table 1. Notations of the proposed mathematical model.

Nomenclature	
Indices	
i	Indices for origin, for all $i = 1,2,\dots,I$
j	Indices for destinations, for all $j = 1,2,\dots,J$
k	Indices for conveyances, for all $k = 1,2,\dots,K$
t	Indices for objective functions, for all $t = 1,2,\dots,T$
Variable Involved	
x_{ijk}	Quantity of product shipped from origin i to destination j by conveyance k
Parameters	
a_i	Product availability at the source i
b_j	Product demand at destination j
e_k	Product shipment capacities of conveyance k
$C_{ijk}^{(t)}$	Objective function coefficients for the objective at level t
$\tilde{a}_i$	Uncertain product availability at i
$\tilde{b}_j$	Uncertain product requirement at j
$\tilde{e}_k$	Uncertain product shipment capacities of conveyance k
$\bar{x}_t$	Decision vector controlled by the t^{th} level DM
α_i	Confidence level for 1st constraint
β_j	Confidence level for 2nd constraint
γ_k	Confidence level for 3rd constraint
$\xi_{ijk}^{(t)}$	Objective function coefficient (Uncertain) for the objective at level t

3 | Formulation of a Mathematical Model with Known Parameters

The mathematical model of multi-level, multi-objective STP is represented as follows:

Level 1

$$\underset{\bar{x}_1}{\mathrm{Min}} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} C_{ijk}^{(1)} x_{ijk}. \tag{1}$$

Level 2

Where $\bar{x}_2$ solves

$$\underset{\bar{x}_2}{\mathrm{Min}} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} C_{ijk}^{(2)} x_{ijk}. \tag{2}$$

Level 3

Where $\bar{x}_t$ solves

$$\underset{\bar{x}_t}{\mathrm{Min}} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} C_{ijk}^{(t)} x_{ijk}, \tag{3}$$

s.t.

$$\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk} \le a_i, \tag{4}$$

$$\sum_{i=1}^{I}\sum_{k=1}^{K} x_{ijk} \ge b_j, \tag{5}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijk} \le e_k, \tag{6}$$

$$x_{ijk} \ge 0, i = 1,2,..,I, j = 1,2,...,J, k = 1,2,...,K. \tag{7}$$

3.1 | Mathematical Model of Expected Constraints Programming

The initial mathematical *Models (1)-(7)* were formulated under the assumption that all parameters involved are known and fixed. However, uncertainty is inevitable in practical scenarios, adding complexity to the model. Due to the uncertainty present in the mathematical model, direct minimization of the model is not feasible. To address this, we utilize the concept of expected-constrained programming proposed by Liu & Liu [11]. By considering the expected values of the objective functions under a set of chance constraints, the original *Models (1)-(7)* are reformulated. The updated model accounts for uncertainty, providing a more realistic representation of the problem and ensuring its practical applicability. The revised mathematical formulation is presented as follows:

Level 4

$$\underset{\bar{x}_1}{\text{Min}} E\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\xi_{ijk}^{(1)}x_{ijk}\right]. \tag{8}$$

Level 5

Where $\bar{x}_2$ solves

$$\underset{\bar{x}_2}{\text{Min}} E\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\xi_{ijk}^{(2)}x_{ijk}\right]. \tag{9}$$

Level 6

Where $\bar{x}_t$ solves

$$\underset{\bar{x}_t}{\text{Min}} E\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\xi_{ijk}^{(t)}x_{ijk}\right], \tag{10}$$

s.t.

$$P\left\{\sum_{j=1}^{J}\sum_{k=1}^{K}x_{ijk}-\tilde{a}_i\le 0\right\}\ge\alpha_i, \tag{11}$$

$$P\left\{\tilde{b}_j-\sum_{i=1}^{I}\sum_{j=1}^{J}x_{ijk}\le 0\right\}\ge\beta_j, \tag{12}$$

$$P\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}x_{ijk}-\tilde{e}_k\le 0\right\}\ge\gamma_k, \tag{13}$$

$$x_{ijk}\ge 0, i=1,2,..,I, j=1,2,...,J, k=1,2,...,K. \tag{14}$$

The *Models (8)-(14)* are formulated using the expected value criterion, optimizing the objective functions while satisfying probabilistic constraints. *Constraint (11)* ensures resource availability with a confidence level of α_i. *Constraint (12)* guarantees demand fulfillment at destinations with a confidence level of β_j. *Constraint (13)* limits conveyance capacities with a confidence level of γ_k.

3.2 | Deterministic Mathematical Model

Let $\Phi_{\xi_{ijk}^{(t)}}$, $\Phi_{\tilde{a}_i}$, $\Phi_{\tilde{b}_j}$, $\Phi_{\tilde{e}_k}$ be the uncertainty distribution of independent uncertain variables $\xi_{ijk}^{(t)}$, $\tilde{a}_i$, $\tilde{b}_j$, $\tilde{e}_k$ correspondingly. Now, it may be assumed that the functions $\Phi_{\xi_{ijk}^{(t)}}$ and $\Phi_{\tilde{b}_j}$ are strictly increasing concerning $\xi_{ijk}^{(t)}$ and $\tilde{b}_j$, respectively, and the functions $\Phi_{\tilde{a}_i}$ and $\Phi_{\tilde{e}_k}$ are strictly decreasing concerning $\tilde{a}_i$ and $\tilde{e}_k$, respectively. Now, the equivalent deterministic mathematical model for uncertain multi-level multi-objective STP *Models (8)-(14)* can be represented as *Models (15)-(21)*.

Level 7

$$\underset{\bar{x}_1}{\text{Min}}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}x_{ijk}\int_0^1\Phi_{\xi_{ijk}^{(1)}}^{-1}(r)dr. \tag{15}$$

Level 8

where $\bar{x}_2$ solves

$$\underset{\bar{x}_2}{\text{Min}} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk} \int_0^1 \Phi^{-1}_{\xi^{(2)}_{ijk}}(r)dr. \tag{16}$$

Level 9

Where $\bar{x}_t$ solves

$$\underset{\bar{x}_t}{\text{Min}} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk} \int_0^1 \Phi^{-1}_{\xi^{(t)}_{ijk}}(r)dr, \tag{17}$$

s.t.

$$\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk} - \Phi^{-1}_{\tilde{a}_i}(1-\alpha_i) \leq 0, \tag{18}$$

$$\Phi^{-1}_{\tilde{b}_j}(\beta_j) - \sum_{i=1}^{I}\sum_{k=1}^{K} x_{ijk} \leq 0, \tag{19}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk} - \Phi^{-1}_{\tilde{e}_k}(1-\gamma_k) \leq 0, \tag{20}$$

$$x_{ijk} \geq 0, i = 1,2,..,I, j = 1,2,...,J, k = 1,2,...,K. \tag{21}$$

Models (15)-(21) discussed above are entirely deterministic. Where $\Phi^{-1}_{\xi^{(t)}_{ijk}}$, $\Phi^{-1}_{\tilde{\alpha}_i}$, $\Phi^{-1}_{\tilde{b}_j}$, and $\Phi^{-1}_{\tilde{e}_k}$ are the inverse uncertainty distributions corresponding to $\xi^{(t)}_{ijk}$, $\tilde{a}_i$, $\tilde{b}_j$, and $\tilde{e}_k$, respectively.

4 | Methodology

Multi-level multi-objective optimization problems are inherently more complex than conventional mathematical models. Abo-Sinna [17] introduced a fuzzy approach to address a two-level multi-objective optimization problem under an uncertain environment, which Osman et al. [18] later expanded to resolve more extensive multi-level problems involving several objectives at each level. This study's multi-level, multi-objective STP is designed with a single objective at each hierarchical level. To solve the above problem, we adopted the method proposed by Osman et al. [18], tailored specifically for single-objective scenarios at each level.

Level 10

$$\underset{\bar{x}_1}{\text{Min}} F_1(x) = f_1(x_{ijk}). \tag{22}$$

Level 11

$$\underset{\bar{x}_2}{\text{Min}} F_2(x) = f_2(x_{ijk}). \tag{23}$$

Level 12

$$\underset{\bar{x}_t}{\text{Min}} F_t(x) = f_t(x_{ijk}), \tag{24}$$

s.t.

Eqs. (18)-(21).

The set of *Eqs. (22)-(24)* represents the general formulation of multi-level programming with 'T' levels of decision-making. The decision variables (x_{ijk}, $i = 1,2,..,I, j = 1,2,...,J, k = 1,2,...,K$) are partitioned among the 'T' levels, where first, second, and 'T' level DM controls the set of decision variables $\bar{x}_1$, $\bar{x}_2$ and $\bar{x}_t$ respectively, whereas $\bigcup_{t=1}^{T} \bar{x}_t = x_{ijk}$ and $\bigcap_{t=1}^{T} \bar{x}_t = 0$. Here, $\bar{x}_t \in R^{p_t} (t = 1,2,..., T)$ is the set of decision variables representing the first, second, and 'T' level DM`s choice, $p_t \geq 1, (t = 1,2,...T)$. $F_1: R^{p_1} \times R^{p_2}...\times R^{p_t} \rightarrow R^{p_1}$ be the first level objective function, $F_2: R^{p_1} \times R^{p_2}...\times R^{p_t} \rightarrow R^{p_2}$ be the second level DM`s objective function, $F_T: R^{p_1} \times R^{p_2}...\times R^{p_T} \rightarrow R^{p_T}$ be the $'T^{th}'$ level objective function. At each level, the objective function is solved individually to find the best compromise solutions. The DM at the first level sets decision variables and goals, allowing some flexibility for the second-level DM to refine the solution. Similarly, each upper-level DM passes their controlled decision variables and goals, with some leeway, to the next level. This process continues until the final-level DM determines a solution that aligns closely with the satisfactory outcomes of the first and all preceding levels.

4.1 | Flexibility on Decision Variables

Let $x^{1*} = \text{Min}F_1(x) = \{x_{ijk}^1 | i = 1,2,...,I, j = 1,2,...,J, k = 1,2,...,K\}$ be the individual best solution obtained for the first level DM when the problem is solved individually. An individual's worst solution for the first-level problem may be obtained as $x^{1-} = \text{Max}F_1(x) = \{x_{ijk}^{1-} | i = 1,2,...,I, j = 1,2,...,J, k = 1,2,...,K\}$. Similarly, Individual best and worst solutions for the second and subsequent levels are obtained and denoted as $x^{2*},...,x^{T*}$ and $x^{2-},...,x^{T-}$ respectively. The solutions of all the levels are disclosed, which are different because of the conflicting nature of the different level objective functions. Using individual best solutions of one level as a solution to another is impractical; therefore, some leeway in the form of tolerances is provided. Tolerances are provided by the higher-level DMs for the decision variables under their control to the lower-level DMs to search for satisfactory solutions in a wider feasible domain. Therefore, the range of each decision variable in the sets $\bar{x}_1$, $\bar{x}_2$,..., and $\bar{x}_{T-1}$ should be around the corresponding decision variable values of the x_{ijk}^1, x_{ijk}^2,..., and x_{ijk}^{T-1}, respectively. $q_1, q_2, ..., q_{T-1}$ are the maximum tolerances associated with the set of decision variables $\bar{x}_1$, $\bar{x}_2$,..., $\bar{x}_{T-1}$ respectively. The membership functions for each of the decision variables under $\bar{x}_1$, $\bar{x}_2$,..., $\bar{x}_{T-1}$ may be formulated as:

$$\mu_{\bar{x}_1} = \begin{cases} \dfrac{\bar{x}_1 - (x^{1*} - q_1)}{q_1}, (x^{1*} - q_1) \leq \bar{x}_1 \leq x^{1*}, \\ \dfrac{(x^{1*} + q_1) - \bar{x}_1}{q_1}, x^{1*} \leq \bar{x}_1 \leq (x^{1*} + q_1). \end{cases} \tag{25}$$

$$\mu_{\bar{x}_{T-1}} = \begin{cases} \dfrac{\bar{x}_{T-1} - \left(x^{(T-1)*} - q_{T-1}\right)}{q_{t-1}}, (x^{(T-1)*} - q_{T-1}) \leq \bar{x}_{T-1} \leq x^{(T-1)*}, \\ \dfrac{\left(x^{(T-1)*} + q_{T-1}\right) - \bar{x}_{T-1}}{q_{T-1}}, x^{(T-1)*} \leq \bar{x}_{T-1} \leq \left(x^{(T-1)*} + q_{T-1}\right). \end{cases} \tag{26}$$

4.2 | Membership Function of the Objectives

Regarding first-level DM's goals and objectives, $f_1(x_{ijk}) \leq f_1^*$ is acceptable, and $f_1(x_{ijk}) > f_1^-$ is absolutely unacceptable. Here, f_1^* and f_1^- are the values of the objective function of first-level DM when the optimal values of first-level and second-level DM are incorporated into the objective function of the first-level DM. Here f_1^- may be obtained as $f_1^- = f_1(x_{ijk}^2)$, where x_{ijk}^2 is the individual the best solution obtained for the second level DM. Membership functions for the goals and objectives of the initial level DM may be formulated as follows:

$$\mu_{f_1}[f_1(x_{ijk})] = \begin{cases} 1 & \text{if } f_1(x_{ijk}) \geq f_1^*, \\ \dfrac{f_1(x_{ijk}) - f_1^-}{f_1^* - f_1^-}, & \text{if } f_1^- \leq f_1(x_{ijk}) \leq f_1^*, \\ 0 & \text{iff}_1(x_{ijk}) \leq f_1^-. \end{cases} \quad (27)$$

Similarly, regarding the goals and the objectives of the follower, $f_2(x_{ijk}) \leq f_2{}^*$ is unconditionally acceptable, and $f_2(x_{ijk}) > f_2{}^-$ is unconditionally unacceptable. Here, $f_2{}^*$ and $f_2{}^-$ are the values of the objective function of the second-level DM when the optimal values of follower and leader are considered for the objective of the second-level decision-maker, respectively. The membership functions for the objective of second-level DM may be formulated as follows:

$$\mu_{f_2}[f_2(x_{ijk})] = \begin{cases} 1 & \text{if } f_2(x_{ijk}) \geq f_2^*. \\ \dfrac{f_2(x_{ijk}) - f_2^-}{f_2^* - f_2^-}, & \text{if } f_2^- \leq f_2(x_{ijk}) \leq f_2^*, \\ 0 & \text{iff}_2(x_{ijk}) \leq f_2^-. \end{cases} \quad (28)$$

Similarly, the membership functions for the objective of the t^{th} level decision-maker may be formulated as follows:

$$\mu_{f_T}[f_T(x_{ijk})] = \begin{cases} 1 & \text{if } f_T(x_{ijk}) \geq f_T^*, \\ \dfrac{f_T(x_{ijk}) - f_T^-}{f_T^* - f_T^-}, & \text{if } f_T^- \leq f_T(x_{ijk}) \leq f_T^*, \\ 0 & \text{iff}_T(x_{ijk}) \leq f_T^-. \end{cases} \quad (29)$$

Here, $f_t{}^*$ and $f_T{}^-$ are the values of the objective function of T^{th} level DM when the optimal values of T^{th} and $(T-1)^{th}$ levels are incorporated into the objective function of T^{th} level DM. Thus, $f_T{}^*$ and $f_T{}^-$ are obtained as $f_T^* = f_T(x_{ijk}^T)$ and $f_T^- = f_T(x_{ijk}^{T-1})$.

4.3 | Global Evolution of the Problem

The overall solution for the multi-level, multi-objective STP, as defined in *Eqs. (15)-(21)*, can be determined by solving the model represented by *Eq. (31)*.

$$\begin{aligned} &\text{Max } \lambda, \\ &\text{s.t.} \\ &\mu_{\tilde{x}_r} \geq \lambda r = 1,2,\ldots,T-1, \\ &\mu_{f_t}[f_t(x_{ijk})] \geq \lambda t = 1,2,\ldots,T, \\ &\lambda \in [0,1]. \end{aligned} \quad (30)$$

Eq. (18)-(21).

Where λ is the maximum degree of satisfaction achieved for all the levels through the generated final solutions.

4.4 | Procedure of the Problem

Step 1. Construct the multi-level, multi-objective decision-making model for uncertain STP as described in Section 2.

Step 2. Transform the model developed in *Step 1* into deterministic form by applying the procedure provided in Section 3.

Step 3. Solve the decision-making problem at each level independently to generate the best solutions for each level.

Step 4. If the solutions of all the levels coincide, stop. The optimal solution has been attained. Otherwise, move to *Step 5*.

Step 5. For t-level STP, create membership functions for the decision variables under the control of first r-level DMs (for all $r = 1,2,\dots,t-1$) using *Eqs. (26)* and *(27)*.

Step 6. Membership functions for goals and objectives are then created for each level using *Eqs. (28)-(30)*.

Step 7. Construct *Model (31)* to generate the final solutions.

Step 8. If the DMs are satisfied with the generated final solutions, stop. Otherwise, move to *Step 9*.

Step 9. Move to *Step 5* to create new membership functions for decision variables using different tolerance values. Repeat the procedure from *Step 6* until a satisfactory solution has been generated.

5 | Numerical Illustration

In a logistics firm, the primary challenge is to design an optimal transportation plan for the STP while addressing three hierarchical objectives: Minimizing transportation cost, total time, and the number of defective items during transit. These objectives are arranged hierarchically, with the first-level DM focused on minimizing costs, the second-level DM aiming to reduce total transportation time, and the third-level DM striving to minimize the number of defective items. The problem involves two conveyance options with specific capacities for transporting products from source points, each with defined supply capacities, to destination points with specified demand requirements. Given the continuous nature of the data, all variables are modeled as follows: The normal distribution. This approach is supported by the central limit theorem, which states that the sampling distribution of many non-normal distributions approaches normality as the sample size increases. The parameters for the numerical illustration are outlined as follows:

$$\begin{cases} \xi_{ijk}^{(1)} \sim N(e_{ijk}^{(1)}, \sigma_{ijk}^{(1)}), \\ \xi_{ijk}^{(2)} \sim N\left(e_{ijk}^{(2)}, \sigma_{ijk}^{(2)}\right), \\ \xi_{ijk}^{(3)} \sim N\left(e_{ijk}^{(3)}, \sigma_{ijk}^{(3)}\right), \end{cases} \qquad i = 1,2,3,4, j = 1,2,3,4, k = 1,2.$$

$$\begin{cases} \tilde{a}_i \sim N(e_{\ddot{\imath}}, \sigma_i), \\ \tilde{b}_j \sim N\left(e'_j, \sigma'_j\right), \\ \tilde{c}_k \sim N(e''_k, \sigma''_k), \end{cases} \qquad i = 1,2,3,4, j = 1,2,3,4, k = 1,2.$$

The data of the uncertain parameters is represented as follows:

Table 2. Transportation costs by conveyance $\mathbf{k_1} \sim N(e_{ij1}^{(1)}, \sigma_{ij1}^{(1)})$.

$N(e_{ij1}^{(1)}, \sigma_{ij1}^{(1)})$	1	2	3	4
1	(20,2)	(18,1.5)	(18,2)	(13,1)
2	(19,1)	(13,1)	(16,1.5)	(18,2)
3	(15,2)	(11,2)	(17,1)	(12,1)
4	(14,1.5)	(14,1.5)	(16,2)	(13,1.5)

Table 3. Transportation costs by conveyance $k_2 \sim N(e_{ij2}^{(1)}, \sigma_{ij2}^{(1)})$.

$N(e_{ij2}^{(1)}, \sigma_{ij2}^{(1)})$	1	2	3	4
1	(30,1)	(35,1.5)	(20,2)	(20,2)
2	(22,1.5)	(27,2)	(27,2)	(23,1)
3	(25,1.5)	(23,1)	(16,2)	(16,2)
4	(30,2)	(21,1)	(26,1.5)	(25,1.5)

Table 4. Transportation time by conveyance $k_1 \sim N(e_{ij1}^{(2)}, \sigma_{ij1}^{(2)})$.

$N(e_{ij1}^{(2)}, \sigma_{ij1}^{(2)})$	1	2	3	4
1	(30,2)	(34,1)	(34,1.5)	(34,1)
2	(26,1.5)	(24,2)	(31,1)	(29,1)
3	(21,1.5)	(20,2)	(26,1.5)	(29,1.5)
4	(21,1.5)	(22,1)	(25,2)	(31,1.5)

Table 5. Transportation time by conveyance $k_2 \sim N(e_{ij2}^{(2)}, \sigma_{ij2}^{(2)})$.

$N(e_{ij2}^{(2)}, \sigma_{ij2}^{(2)})$	1	2	3	4
1	(38,1)	(38,1)	(32,1.5)	(30,2)
2	(25,1.5)	(28,2)	(28,2)	(28,1.5)
3	(29,1)	(33,1.5)	(34,2)	(22,1)
4	(30,2)	(28,2)	(24,1.5)	(24,1.5)

Table 6. Number of defected items by conveyance $k_1 \sim N(e_{ij1}^{(3)}, \sigma_{ij1}^{(3)})$.

$N(e_{ij1}^{(3)}, \sigma_{ij1}^{(3)})$	1	2	3	4
1	(9,1.5)	(10,1)	(8,1)	(6,1)
2	(8,1)	(11,1.5)	(7,1.5)	(7,1)
3	(11,2)	(10,1.5)	(7,2)	(11,1.5)
4	(11,1.5)	(10,1.5)	(10,1)	(12,1.5)

Table 7. Number of defected items by conveyance $k_2 \sim N(e_{ij2}^{(3)}, \sigma_{ij2}^{(3)})$.

$N(e_{ij2}^{(3)}, \sigma_{ij2}^{(3)})$	1	2	3	4
1	(7,2)	(10,1)	(9,1)	(8,1.5)
2	(9,1.5)	(8,1.5)	(7,1)	(11,1.5)
3	(9,1.5)	(8,1.5)	(11,1.5)	(11,1)
4	(8,1)	(7,2)	(10,2)	(10,1)

Table 8. Supply availability parameters at origins $\sim N(e_i, \sigma_i)$.

$N(e_i, \sigma_i)$	1	2	3	4
	(25,1.5)	(30,1.5)	(32,2)	(28,2)

Table 9. Demand parameters at the destinations $\sim N(e_j', \sigma_j')$.

$N(e_i', \sigma_i')$	1	2	3	4
	(10,1.5)	(14,1)	(22,1)	(18,1)

Table 10. Parameters of normal distribution $N(e''_k, \sigma''_k)$ of transportation capacities.

$N(e''_i, \sigma''_i)$	1	2
	(40,1.5)	(60,1)

5.1 | Mathematical Model of the Proposed Problem

As all the parameters defined in the problem follow a normal distribution, the function for the inverse normal distribution is used to model the numerical problem. The equivalent deterministic model for this problem is:

Level 13

$$\underset{\bar{x}_1}{\text{Min}} \sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{k=1}^{2} x_{ijk} e_{ijk}^{(1)}. \tag{31}$$

Level 14

$$\underset{\bar{x}_2}{\text{Min}} \sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{k=1}^{2} x_{ijk} e_{ijk}^{(2)}. \tag{32}$$

Level 15

$$\underset{\bar{x}_3}{\text{Min}} \sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{k=1}^{2} x_{ijk} e_{ijk}^{(3)}, \tag{33}$$

s.t.

$$\sum_{j=1}^{4} \sum_{k=1}^{2} x_{ijk} - \left[e_i + \frac{\sigma_i \sqrt{3}}{\pi} \ln \frac{1-\alpha_i}{\alpha_i} \right] \leq 0, i = 1,2,3,4, \tag{34}$$

$$\left[e'_j + \frac{\sigma'_j \sqrt{3}}{\pi} \ln \frac{\beta_j}{1-\beta_j} \right] - \sum_{i=1}^{4} \sum_{k=1}^{2} x_{ijk} \leq 0, j = 1,2,3,4, \tag{35}$$

$$\sum_{i=1}^{4} \sum_{j=1}^{4} x_{ijk} - \left[e''_k + \frac{\sigma''_k \sqrt{3}}{\pi} \ln \frac{1-\gamma_k}{\gamma_k} \right] \leq 0, k = 1,2, \tag{36}$$

$$x_{ijk} \geq 0, i = 1,2,3,4, j = 1,2,3,4, k = 1,2, \tag{37}$$

where

$\bar{x}_1 = \{x_{111}, x_{121}, x_{131}, x_{141}, x_{211}, x_{221}, x_{231}, x_{241}, x_{311}, x_{321}, x_{331}, x_{341}\}$,

decision variables that are to be controlled by I-level DM.

$\bar{x}_2 = \{x_{411}, x_{421}, x_{431}, x_{441}, x_{112}, x_{122}, x_{132}, x_{142}\}$,

decision variables that are to be controlled by II-level DM.

$\bar{x}_1 = \{x_{212}, x_{222}, x_{232}, x_{242}, x_{312}, x_{322}, x_{332}, x_{342}, x_{412}, x_{422}, x_{432}, x_{442}\}$,

decision variables that are to be controlled by the III level DM.

6 | Result

After putting the values of all the parameters from the data in the *Models (32)-(37)*, we obtain a simple, crisp tri-level multi-objective STP. First, individual best solutions are obtained for the objectives at each level. The individual best solutions for each objective (When solved independently) are given in *Table 11*. The solutions obtained for the individual objectives at each level are different. To reach the compromise solution, membership functions $(\mu_{\bar{x}_r}, r = 1,2)$ with suitable tolerances are created using *Eqs. (26)* and *(27)* for the decision variables controlled by the 1 and 2 level DM. Membership functions $(\mu_{f_T}, T = 1,2,3)$ for each level, objective functions are also created according to the *Eqs. (28)-(30)*.

Table 11. Individual best solutions for each objective.

Objective Functions at Three Levels	Individual Best Solution
Level 10 (f_1)	1015.254
Level 11 (f_2)	1589.299
Level 12 (f_3)	474.793

Now, assuming λ as the maximum degree of satisfaction achieved for all the levels, the final model is then formulated as per *Model (31)*:

$$
\begin{aligned}
&\text{Max } \lambda,\\
&\text{s.t.}\\
&\mu_{\bar{x}_r} \geq \lambda r = 1,2,\\
&\mu_{f_T}\left[f_T\left(x_{ijk}\right)\right] \geq \lambda T = 1,2,3,\\
&\lambda \in [0,1].
\end{aligned} \tag{38}
$$

Eq. (18)-(21) Where f_T is the objective function at the T^{th} level.

The final *Model (38)* obtained is a well-defined Tchebycheff problem, giving a Pareto optimal STP solution. The confidence levels for the chance constraints are assumed to be $\alpha_i = 0.9$, $\beta_j = 0.9$, and $\gamma_k = 0.9$. Using the Lingo 20.0 optimization software, the Tchebycheff *Model (39)* is solved, and the results are given in *Table 12*. If the DM at the first level is convinced by the results obtained, then the above result is a convincing solution. Otherwise, the first-level DM should provide or try for some other tolerances and present new membership functions to the subsequent level DM. Similarly, the second-level DM should give a new membership function for the decision variables and objectives to the DM at the third level. This process continues until a convincing result is obtained.

Table 12. Compromise on an optimal solution for the model.

Optimal Transportation Policy	Objective Function Values		
	Level I $(\mathbf{Z}_1^*)$	Level II $(\mathbf{Z}_2^*)$	Level III $(\mathbf{Z}_3^*)$
x_{141}=10.6544, x_{221}=15.2114, x_{241}=0.50, x_{341}=0.50, x_{411}=11.3171, x_{112}=0.50, x_{232}=8.1826, x_{242}=4.2889, x_{332}=15.0288, x_{342}=2.7680	1139.020	1949.062	672.4620

7 | Conclusion

In this study, we developed a hierarchical, multi-level, multi-objective mathematical model for solving the STP under uncertain environments. The proposed model is designed to reflect the practical challenges encountered by logistics and supply chain firms, where multiple conflicting objectives and uncertain parameters must be addressed concurrently. By introducing three hierarchical objectives—minimization of transportation cost, minimization of transportation time, and minimization of defective items—the model captures the essential trade-offs involved in efficient and reliable transportation planning. One of the key contributions of this work lies in the incorporation of normal uncertain variables, which allows the model to realistically represent fluctuations in transportation parameters such as supply, demand, cost, and delivery time. Leveraging the central limit theorem, we model these uncertainties in a mathematically sound manner, thereby enhancing the robustness and reliability of the decision-making process.

The model is further strengthened by employing a fuzzy logic-based approach, which ensures the derivation of acceptable and coherent solutions at each level of the hierarchy. This approach enables decision-makers to handle ambiguity and vagueness in real-world scenarios while maintaining consistency across different levels of decision-making. The hierarchical structure provides a strategic advantage by enabling upper-level goals to guide and constrain lower-level decisions, ensuring alignment with organizational priorities. Through a comprehensive numerical illustration, the practical applicability and efficiency of the proposed model have been demonstrated. The results indicate that the model effectively balances multiple objectives while coping with uncertainties, offering flexible and adaptive solutions. It supports informed trade-off analysis, helping stakeholders to prioritize goals based on operational needs and constraints. The proposed multi-level, multi-objective model under uncertain environments provides a powerful decision-support tool for optimizing solid transportation systems. It contributes significantly to the fields of operations research and logistics by addressing the complexities of real-world TPs. Future research may explore the integration of more sophisticated uncertainty modeling techniques, hybrid optimization algorithms, or real-time data analytics to enhance further the model's adaptability and performance in dynamic environments.

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Authors' Contributions

WA conceptualized the study, developed the mathematical models, and wrote the manuscript. MSJ and MN conducted a numerical experiment, analyzed results, and prepared visual representations. WA supervised the research, ensured mathematical rigor, and refined the manuscript.

Data Availability

Required data is available in this manuscript.

Conflict of Interest

There are no competing interests to declare.

Consent for Publication

All authors have provided their consent for the publication of this manuscript.

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