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Discussion on Entropy and Similarity Measures and Their Few Applications Because of Vague Soft Sets

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Abstract

This study focuses on two crucial techniques that are used for measurements. These techniques are introduced in vague soft sets concerning crisp points of the space. Relevant examples support these strategies.

Keywords: Vague soft set, Entropy measure, Similarity measure, Interval vague soft sets.

1 | Introduction

Many attempts to model confusing data using traditional mathematics may fail. This is the case because the concept of uncertainty is too convoluted and poorly defined. As a result, a number of theories have been developed to deal with ambiguity and uncertainty. Zadeh [1] proposed the concept of fuzzy set theory as the first attempt in this field FST. Because it primarily addressed the membership function, this theory was limited to a few areas of mathematics and had flaws. It has nothing to do with the function of non-membership. As a result, a broad generalization was required, and Atanassov [2] proposed the concept of Intuitionistic Fuzzy Set Theory (IFST). The membership and non-membership functions are addressed in this theory. To apply IFST to medical diagnosis, De et al. [3] employed three steps: determining symptoms, building medical knowledge totally based on Intuitionistic Fuzzy Relations (IFR), and establishing diagnosis based on the composition of symptoms. Sales analysis, new product promotion, financial services, negotiation, and even psychological studies can all be represented using this approach. Mathematicians worked tirelessly to demonstrate its practical uses, and progress was achieved in this regard.

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Atanassov [4, 5] applied the concept of distance measurements between Intuitionistic Fuzzy Sets (IFSs) to various practical problems, such as pattern recognition and machine learning. This approach must be modified because FST and IFST provide out-of-error results due to insufficient parameters or decision-making factors. In an intuitionistic fuzzy environment, Garg [6] adjusted certain cosine for IFSs by taking into consideration the connectedness between the pair of membership degrees and offering a Multi-Criteria Decision-Making (MCDM) method based on similarity measure. Liu et al. [7] developed the Cosine Similarity Measure (CSM) between hybrids IFSs and investigated its use in medical diagnosis.

Chen [8] proposed various approaches for dealing with fuzzy decision-making difficulties and demonstrated their use in medical diagnostics. Conversely, these strategies are not constrained and can be used to solve any decision-making challenge. All of the methodologies and theories outlined above had a fundamental flaw: they lacked sufficient parameters. Work on filling this large gap persisted, and eventually, a Russian researcher came up with a new theory known as soft set theory, which became very famous in both pure and applied mathematics. Soft set theory, a comprehensive mathematical instrument for dealing with uncertain circumstances, was first introduced by Molodtsov [9]. By applying soft set theory to a variety of practical issues, Maji et al. [10] increased its utility. Maji et al. [11] filled in the gaps left by [9] and outlined some additional fundamentals in this area. Roy and Maji [12] pioneered some real-world uses of soft set theory. Majumdar and Samanta [13] presented two types of soft-set similarity assessments and used these methods to conduct compare-and-contrast research. Soft set similarity measures and intuitionistic fuzzy soft set similarity measures were installed by Majumdar and Samanta [13]. Bo et al. suggested the concept of similarity measures between two ambiguous soft sets, discussed real-world issues in landmark preferences, and reported some more findings. Hu et al. [14] looked at imprecise soft-set uncertainty measures. They offered them an axiomatic definition of similarity and entropy. A. Kharal [15] looked at the deep similarity measures between two soft sets and found specific faults in [13] that he addressed in this study. Based on these ideas, Mehmood et al. [16] reworked the key conceptions for vague soft sets and created a novel approach for vague soft bi-topological spaces employing ambiguous soft points of the space.

2 | Overview of Vague Soft Sets and Entropy Measures on Vague Soft Sets

This section contains basic definitions that are required for the following sections.

Definition 1 ([16]). Let $\mathbb{U}$ stand for the key set and $\mathbb{E}$ for the parameters. Let $\mathrm{P}(\mathbb{U})$ signifies power set of all vague sets on $\mathbb{U}$. Then, a vague soft set $\langle \tilde{\mathcal{Q}}, \mathbb{E}\rangle$ over $\mathbb{U}$ is a set defined by a set valued function $\tilde{\mathcal{Q}}$ representing a mapping $\tilde{\mathcal{Q}}: \mathbb{E} \to \mathrm{P}(\mathbb{U})$ where $\tilde{\mathcal{Q}}$ is referred to as function of the vague soft set $\langle \tilde{\mathcal{Q}}, \mathbb{E}\rangle$. It can be written as a set of ordered pairs $\langle \tilde{\mathcal{Q}}, \mathbb{E}\rangle = \left[\left(\mathfrak{s}, \langle \varkappa, \mathbb{T}_{\tilde{\mathcal{Q}}(\mathfrak{s})(\varkappa)}, \mathbb{F}_{\tilde{\mathcal{Q}}(\mathfrak{s})(\varkappa)} : \varkappa \in \mathbb{U}\rangle\right) : \mathfrak{s} \in \mathbb{E}\right]$, Where $\mathbb{T}_{\tilde{\mathcal{Q}}(\mathfrak{s})(\varkappa)}, \mathbb{F}_{\tilde{\mathcal{Q}}(\mathfrak{s})(\varkappa)} \in [0,1]$, respectively called the-membership and the non-membership function of $\tilde{\mathcal{Q}}(\mathfrak{s})$. Since supremum of each function is 1 and infimum of each function is o so the inequality $0 \leq \mathbb{T}_{\tilde{\mathcal{Q}}(\mathfrak{s})(\varkappa)} + \mathbb{F}_{\tilde{\mathcal{Q}}(\mathfrak{s})(\varkappa)} \leq 2$ is obviously true.

Definition 2 ([16]). Let $\langle \tilde{\mathcal{Q}}, \mathbb{E}\rangle$ be a (VSS) over the key set $\mathbb{U}$ then complement of $\langle \tilde{\mathcal{Q}}, \mathbb{E}\rangle$ is symbolized by $\langle \tilde{\mathcal{Q}}, \mathbb{E}\rangle^{c}$ and is defined as follows:

$$\langle \tilde{\mathcal{Q}}, \mathbb{E}\rangle^{c} = \left[\left(\mathfrak{s}, \langle \varkappa, \mathbb{F}_{\tilde{\mathcal{Q}}(\mathfrak{s})(\varkappa)}, \mathbb{T}_{\tilde{\mathcal{Q}}(\mathfrak{s})(\varkappa)} : \varkappa \in \mathbb{U}\rangle\right) : \mathfrak{s} \in \mathbb{E}\right],$$

$$\left(\langle \tilde{\mathcal{Q}}, \mathbb{E}\rangle^{c}\right)^{c} = \langle \tilde{\mathcal{Q}}, \mathbb{E}\rangle .$$

Definition 3 ([16]). Let $\langle \widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle$ and $\langle \widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle$ two (VSSs) over the key set $\mathbb{U}$ then $\langle \widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle \widetilde{\subseteq} \langle \widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle$ if $\mathbb{T}_{\widetilde{\mathcal{Q}_1}(\mathfrak{s})(\varkappa)} \preccurlyeq \mathbb{T}_{\widetilde{\mathcal{Q}_2}(\mathfrak{s})(\varkappa)}, \mathbb{F}_{\widetilde{\mathcal{Q}_1}(\mathfrak{s})(\varkappa)} \succcurlyeq \mathbb{F}_{\widetilde{\mathcal{Q}_2}(\mathfrak{s})(\varkappa)}$, for all $\mathfrak{s} \in \mathbb{E}$ and for all $\varkappa \in \mathbb{U}$. If $\langle \widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle \widetilde{\subseteq} \langle \widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle$ and $\langle \widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle \widetilde{\supseteq} \langle \widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle$, then $\langle \widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle = \langle \widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle$.

Definition 4 ([16]). Let $\langle \widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle \widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle$ be two (VSSs) over key set $\mathbb{U}$ such that $\langle \widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle \neq \langle \widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle$ then their union is denoted by $\langle \widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle \tilde{\cup} \langle \widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle = (\widetilde{\mathcal{Q}_3}, \mathbb{E})$ and is defined as

$$(\widetilde{\mathcal{Q}_3}, \mathbb{E}) = \left[\left(\mathfrak{s}, \langle \varkappa, \mathbb{T}_{\widetilde{\mathcal{Q}_3}(\mathfrak{s})(\varkappa)}, \mathbb{F}_{\widetilde{\mathcal{Q}_3}(\mathfrak{s})(\varkappa)} : \varkappa \in \pi\rangle\right) : \mathfrak{s} \in \mathbb{E}\right],$$

where

$\mathbb{T}_{\widetilde{Q_3}(\mathfrak{s})}(\varkappa) = max\left[\mathbb{T}_{\widetilde{Q_1}(\mathfrak{s})}(\varkappa), \mathbb{T}_{Q_2(\mathfrak{s})}(\varkappa)\right],$

$\mathbb{F}_{\widetilde{Q_3}(\mathfrak{s})}(\varkappa) = min\left[\mathbb{F}_{\widetilde{Q_1}(\mathfrak{s})}(\varkappa), \mathbb{F}_{Q_2(\mathfrak{s})}(\varkappa)\right].$

Definition 5 ([16]). Let $\langle\widetilde{Q_1}, \mathbb{E}\rangle, (\tilde{G}, \mathbb{E})$ be two (VSSs) over key set $\mathbb{U}$ such that $\langle\widetilde{Q_1}, \mathbb{E}\rangle \neq (\widetilde{Q_2}, \mathbb{E})$ then their intersection is denoted by $\langle\widetilde{Q_1}, \mathbb{E}\rangle \tilde{\cap} (\tilde{G}, \mathbb{E}) = (\widetilde{Q_3}, \mathbb{E})$ and is defined as

$$(\widetilde{Q_3}, \mathbb{E}) = \left[\left(\mathfrak{s}, \langle\varkappa, \mathbb{T}_{\widetilde{Q_3}(\mathfrak{s})}(\varkappa), \mathbb{F}_{\widetilde{Q_3}(\mathfrak{s})}(\varkappa) : \varkappa \in \mathbb{U}\rangle\right) : \mathfrak{s} \in \mathbb{E}\right],$$

where

$$(\widetilde{Q_3}, \mathbb{E}) = \left[\left(\mathfrak{s}, \langle\varkappa, \mathbb{T}_{\widetilde{Q_3}(\mathfrak{s})}(\varkappa), \mathbb{F}_{\widetilde{Q_3}(\mathfrak{s})}(\varkappa) : \varkappa \in \mathbb{U}\rangle\right) : \mathfrak{s} \in \mathbb{E}\right],$$

$\mathbb{T}_{Q_3(\mathfrak{s})}(\varkappa) = min\left[\mathbb{T}_{\widetilde{Q_1}(\mathfrak{s})}(\varkappa), \mathbb{T}_{\widetilde{Q_2}(\mathfrak{s})}(\varkappa)\right],$

$\mathbb{F}_{\widetilde{Q_3}(\mathfrak{s})}(\varkappa) = max\left[\mathbb{F}_{\widetilde{Q_1}(\mathfrak{s})}(\varkappa), \mathbb{F}_{\widetilde{Q_2}(\mathfrak{s})}(\varkappa)\right].$

Definition 6. Let $H: VSS(\mathbb{U}) \rightarrow [0,1]$ be a mapping, where $VSS(\mathbb{U})$ denotes the set of all (VSSs) on $\mathbb{U}$. For $\langle\widetilde{Q_1}, \mathbb{E}\rangle \in VSS(U), H\langle\widetilde{Q_1}, \mathbb{E}\rangle$ is called the entropy of if it satisfies the following conditions:

I. $H\langle\widetilde{Q_1}, \mathbb{E}\rangle = 0 \Leftrightarrow$ for all $\mathfrak{s} \in \mathbb{E}, \varkappa \in \mathbb{U}, \mathbb{T}_{Q_1(\mathfrak{s})}(\varkappa) = 0$ and $\mathbb{F}_{Q_1(\mathfrak{s})}(\varkappa) = 1$ or $\mathbb{T}_{Q_1(\mathfrak{s})}(\varkappa) = 1$ and $\mathbb{F}_{Q_1(\mathfrak{s})}(\varkappa) = 0$.

II. $H\langle\widetilde{Q_1}, \mathbb{E}\rangle = 1$ for all $\mathfrak{s} \in E, \varkappa \in \mathbb{U}, \mathbb{T}_{Q_1(\mathfrak{s})}(\varkappa) = \mathbb{F}_{Q_1(\mathfrak{s})}(\varkappa) = 0.5,$

III. $H\langle\widetilde{Q_1}, \mathbb{E}\rangle = H\langle\widetilde{Q_1}, \mathbb{E}\rangle^c,$

IV. for all $\mathfrak{s} \in \mathbb{E}, \varkappa \in \mathbb{U}$ when $\langle\widetilde{Q_1}, \mathbb{E}\rangle \subseteq \langle\widetilde{Q_2}, \mathbb{E}\rangle$ and $\mathbb{T}_{\widetilde{Q_2}(\mathfrak{s})}(\varkappa) \leq \mathbb{F}_{Q_2(\mathfrak{s})}(\varkappa)$, or $\langle\widetilde{Q_1}, \mathbb{E}\rangle \supseteq \langle\widetilde{Q_2}, \mathbb{E}\rangle$, and $\mathbb{T}_{\widetilde{Q_2}(\mathfrak{s})}(\varkappa) \geq \mathbb{F}_{\widetilde{Q_2}(\mathfrak{s})}(\varkappa)$, then $H\langle\widetilde{Q_1}, \mathbb{E}\rangle \leq H\langle\widetilde{Q_2}, \mathbb{E}\rangle$.

Definition 7. Let $d: VSS(\mathbb{U}) \times VSS(\mathbb{U}) \rightarrow [0,1]$ be a mapping for $\langle\widetilde{Q_1}, \mathbb{E}\rangle, \langle\widetilde{Q_2}, \mathbb{E}\rangle \in NSS(U), d(\langle\widetilde{Q_1}, \mathbb{E}\rangle, \langle\widetilde{Q_2}, \mathbb{E}\rangle)$ is called the degree of distance between $\langle\widetilde{Q_1}, \mathbb{E}\rangle$ and $\langle\widetilde{Q_2}, \mathbb{E}\rangle$ if it satisfies the following conditions:

I. $d(\langle\widetilde{Q_1}, \mathbb{E}\rangle, \langle\widetilde{Q_2}, \mathbb{E}\rangle) = d(\langle\widetilde{Q_2}, \mathbb{E}\rangle, \langle\widetilde{Q_1}, \mathbb{E}\rangle).$

II. $d(\langle\widetilde{Q_1}, \mathbb{E}\rangle, \langle\widetilde{Q_2}, \mathbb{E}\rangle) \in [0,1].$

III. $d(\langle\widetilde{Q_1}, \mathbb{E}\rangle, \langle\widetilde{Q_2}, \mathbb{E}\rangle) = 1 \Leftrightarrow$ for all $\mathfrak{s} \in \mathbb{E}, \varkappa \in \mathbb{U}, \mathbb{T}_{\widetilde{Q_1}(\mathfrak{s})}(\varkappa) = 0.$

IV. $\mathbb{F}_{\widetilde{Q_1}(\mathfrak{s})}(\varkappa) = 1,$ and $\mathbb{T}_{\widetilde{Q_2}(\mathfrak{s})}(\varkappa) = 1,$
$\mathbb{F}_{\widetilde{Q_2}(\mathfrak{s})}(\varkappa) = 0,$ or $\mathbb{T}_{\widetilde{Q_1}(\mathfrak{s})}(\varkappa) = 1,$
$\mathbb{F}_{\widetilde{Q_1}(\mathfrak{s})}(\varkappa) = 0,$ or $\mathbb{T}_{\widetilde{Q_2}(\mathfrak{s})}(\varkappa) = 0,$
$\mathbb{F}_{\widetilde{Q_2}(\mathfrak{s})}(x) = 1.$

V. $d(\langle\widetilde{Q_1}, \mathbb{E}\rangle, (\langle\widetilde{Q_2}, \mathbb{E}\rangle)) = 0 \Leftrightarrow \langle\widetilde{Q_1}, \mathbb{E}\rangle = \langle\widetilde{Q_2}, \mathbb{E}\rangle.$

VI. $\langle\widetilde{Q_1}, \mathbb{E}\rangle \subseteq \langle\widetilde{Q_2}, \mathbb{E}\rangle \subseteq \langle\widetilde{Q_3}, \mathbb{E}\rangle \Rightarrow d(\langle\widetilde{Q_1}, \mathbb{E}\rangle, \langle\widetilde{Q_3}, \mathbb{E}\rangle) \geq max(d(\langle\widetilde{Q_1}, \mathbb{E}\rangle, \langle\widetilde{Q_2}, \mathbb{E}\rangle), d(\langle\widetilde{Q_2}, \mathbb{E}\rangle, \langle\widetilde{Q_3}, \mathbb{E}\rangle)), \langle\widetilde{Q_3}, \mathbb{E}\rangle \in VSS(\mathbb{U}).$

Definition 8. Let $\mathbb{U} = \{\varkappa_1, \varkappa_2, \dots \dots \varkappa_n\}$ be the universal set of elements and $\mathbb{E} = \{\mathfrak{s}_1, \mathfrak{s}_2, \dots \dots \dots \mathfrak{s}_m\}$ be the universal set of parameters, then we have

$$H\langle\widetilde{Q_1}, \mathbb{E}\rangle = \frac{1}{m}\sum_{i=1}^{n} H_i\langle\widetilde{Q_1}, \mathbb{E}\rangle,$$

where

$$H_i\langle\widetilde{Q_1},\mathbb{E}\rangle = \frac{1}{n}\frac{max\sum_j \text{count}\left(\langle\widetilde{Q_1},\mathbb{E}\rangle \cap \langle\widetilde{Q_1},\mathbb{E}\rangle^c\right)}{max\sum_j \text{count}\left(\langle\widetilde{Q_1},\mathbb{E}\rangle \cup \langle\widetilde{Q_1},\mathbb{E}\rangle^c\right)},$$

$$\max\sum_j \text{count}\left(\langle\widetilde{Q_1},\mathbb{E}\rangle \cap \langle\widetilde{Q_1},\mathbb{E}\rangle^c\right),$$

$$= \sum_{j=1}^{n}\left(\mathbb{T}_{(\langle\widetilde{Q_1},\mathbb{E}\rangle\cap\langle\widetilde{Q_1},\mathbb{E}\rangle^c)(\mathfrak{s}_i)}(\varkappa_j)\right),$$

$$\max\sum_j \text{count}\left(\langle\widetilde{Q_1},\mathbb{E}\rangle \cup \langle\widetilde{Q_1},\mathbb{E}\rangle^c\right),$$

$$= \sum_{j=1}^{n}\left(\mathbb{T}_{(\langle\widetilde{Q_1},\mathbb{E}\rangle\cup\langle\widetilde{Q_1},\mathbb{E}\rangle^c)(\mathfrak{s}_i)}(\varkappa_j)\right),$$

is the entropy of (VSSs).

Definition 9. Let $\mathbb{U} = \{\varkappa_1, \varkappa_2, \dots \varkappa_n\}$ be the universal set elements and $\mathbb{E} = \{\mathfrak{s}_1, \mathfrak{s}_2, \dots \dots \mathfrak{s}_m\}$ be the universal set of parameters; Then, we define normalized Euclidean distance based on Hausdorff metric as $d_1(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle) = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} max\left(\left|\mathbb{T}_{Q_1(\mathfrak{s}_i)}(\varkappa_j) - \mathbb{T}_{Q_2(\mathfrak{s}_i)}(\varkappa_j)\right|, \left|\mathbb{F}_{Q_1(\mathfrak{s}_i)}(\varkappa_j) - \mathbb{F}_{Q_2(\mathfrak{s}_i)}(\varkappa_j)\right|\right)$ and normalized Hamming distance based on Hausdorff metric as

$$d_2((\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle) =$$
$$\left\{\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} max\left(\left(\mathbb{T}_{Q_1(\mathfrak{s}_i)}(\varkappa_j) - \mathbb{T}_{Q_2(\mathfrak{s}_i)}(\varkappa_j)\right)^2, \left(\mathbb{F}_{Q_1(\mathfrak{s}_i)}(\varkappa_j) - \mathbb{F}_{Q_2(\mathfrak{s}_i)}(\varkappa_j)\right)^2\right)\right\}^{\frac{1}{2}}.$$

Theorem 1. Let $\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle$ $\langle\widetilde{Q_3},\mathbb{E}\rangle$ be (VSSs) over $\mathbb{U}$, then distance measure $d_i(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle)$ for $i = 1,2$ between $\langle\widetilde{Q_1},\mathbb{E}\rangle$ and $\langle\widetilde{Q_2},\mathbb{E}\rangle$ Satisfies the following properties.

I. $0 \le d_i(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle) \le 1$.

II. $d_i(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle) = 0$ if and only if $\langle\widetilde{Q_1},\mathbb{E}\rangle = \langle\widetilde{Q_2},\mathbb{E}\rangle$.

III. $d_i(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle) = d_i(\langle\widetilde{Q_2},\mathbb{E}\rangle, \langle\widetilde{Q_1},\mathbb{E}\rangle)$.

IV. If $Q_1 \subseteq Q_2 \subseteq Q_3$, then $d_i(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle) \le d_i(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_3},\mathbb{E}\rangle)$ and $d_i(\langle\widetilde{Q_2},\mathbb{E}\rangle, \langle\widetilde{Q_3},\mathbb{E}\rangle) \le d_i(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_3},\mathbb{E}\rangle)$.

Proof.

I. As membership and non-membership functions lies between 0 and 1, the distance measure based on these function also lies between 0 to 1.

II. If $d_i(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle) = 0$ implies $\left|\mathbb{T}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{T}_{Q_2}(\mathfrak{s}_i)(\varkappa_j)\right| = 0$
$\left|\mathbb{F}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{F}_{Q_2}(\mathfrak{s}_i)(\varkappa_j)\right| = 0$ implies $\mathbb{T}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) = \mathbb{T}_{Q_2}(\mathfrak{s}_i)(\varkappa_j)$, $\mathbb{F}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) = \mathbb{F}_{Q_2}(\mathfrak{s}_i)(\varkappa_j)$,
i.e., $\langle\widetilde{Q_1},\mathbb{E}\rangle = (\langle\widetilde{Q_2},\mathbb{E}\rangle$.
Conversely, Let $\langle\widetilde{Q_1},\mathbb{E}\rangle = \langle\widetilde{Q_2},\mathbb{E}\rangle$, implies $\mathbb{T}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) = \mathbb{T}_{Q_2}(\mathfrak{s}_i)(\varkappa_j)$, $\mathbb{F}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) = \mathbb{F}_{Q_2}(\mathfrak{s}_i)(\varkappa_j)$,
implies $\left|\mathbb{T}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{T}_{Q_2}(\mathfrak{s}_i)(\varkappa_j)\right| = |\mathbb{F}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{F}_{Q_2}(\mathfrak{s}_i)(\varkappa_j)| = 0$ i.e. $d_i(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle) = 0$.

III. Clearly $d(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle) = d(\langle\widetilde{Q_2},\mathbb{E}\rangle, \langle\widetilde{Q_1},\mathbb{E}\rangle)$.

IV. Let $(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle) = max(|\mathbb{T}_{Q_1(\mathfrak{s}_i)}(\varkappa_j) - \mathbb{T}_{Q_2(\mathfrak{s}_i)}(\varkappa_j)|, |\mathbb{F}_{Q_1(\mathfrak{s}_i)}(\varkappa_j) -, \mathbb{F}_{Q_2(\mathfrak{s}_i)}(\varkappa_j)|)$.
$Q_1 \subseteq Q_2 \subseteq Q_3$ implies

$\mathbb{T}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) \le \mathbb{T}_{Q_2}(\mathfrak{s}_i)(\varkappa_j) \le \mathbb{T}_{Q_3}(\mathfrak{s}_i)(\varkappa_j)$,

$\mathbb{F}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) \ge \mathbb{F}_{Q_2}(\mathfrak{s}_i)(\varkappa_j) \ge \mathbb{F}_{Q_3}(\mathfrak{s}_i)(\varkappa_j)$,

To Prove: $d_1(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle) \le d_1(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_3},\mathbb{E}\rangle)$ and $d_1(\langle\widetilde{Q_2},\mathbb{E}\rangle, \langle\widetilde{Q_3},\mathbb{E}\rangle) \le d_1(\langle\widetilde{Q_1},\mathbb{E}\rangle, (\langle\widetilde{Q_3},\mathbb{E}\rangle)$.

Case 1: If $\left|\mathbb{T}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{T}_{Q_3}(\mathfrak{s}_i)(\varkappa_j)\right| \ge \left|\mathbb{F}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{F}_{Q_3}(\mathfrak{s}_i)(\varkappa_j)\right|$ then $D(\langle\widetilde{Q_1},\mathbb{E}\rangle, \langle\widetilde{Q_2},\mathbb{E}\rangle) = \left|\mathbb{T}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{T}_{Q_3}(e_i)(u_j)\right|$.

I. $\left|\mathbb{F}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{F}_{Q_2}(\mathfrak{s}_i)(\varkappa_j)\right| \le |\mathbb{F}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{F}_{Q_3}(\mathfrak{s}_i)(\varkappa_j)| \le |\mathbb{T}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{T}_{Q_3}(\mathfrak{s}_i)(\varkappa_j)|$, for all i and j

II. $\left|\mathbb{F}_{Q_2}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{F}_{Q_3}(\mathfrak{s}_i)(\varkappa_j)\right| \le |\mathbb{F}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{F}_{Q_3}(\mathfrak{s}_i)(\varkappa_j)| \le |\mathbb{T}_{Q_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{T}_{Q_3}(\mathfrak{s}_i)(\varkappa_j)|$, for all i and j

III. $|\mathbb{T}_{\mathcal{Q}_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{T}_{\mathcal{Q}_2}(\mathfrak{s}_i)(\varkappa_j)| \leq | \mathbb{T}_{\mathcal{Q}_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{T}_{\mathcal{Q}_3}(\mathfrak{s}_i)(\varkappa_j)|$, $|\mathbb{T}_{\mathcal{Q}_2}(\mathfrak{s}_i)(u_j) - \mathbb{T}_{\mathcal{Q}_3}(\mathfrak{s}_i)(\varkappa_j)| \leq | \mathbb{T}_{\mathcal{Q}_1}(\mathfrak{s}_i)(\varkappa_j) - \mathbb{T}_{\mathcal{Q}_3}(\mathfrak{s}_i)(\varkappa_j)|$, for all i and j.

Combining (I)-(III) we have $D(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle) \leq D(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$ and $D(\langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle) \leq D(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, (\langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$. Therefore $d_1(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, (\langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle) \leq d_1(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$ and $d_1(\langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle) \leq d_1(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$.

Case 2: We can easily prove that we have $D(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle) \leq D(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$ and $D((\langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$ $D(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$. Therefore $d_1(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle) \leq d_1(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$ and $d_1(\langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle) \leq d_1(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$.

Case 3: Similar to the above cases.

Hence from Cases 1-3 we have if $\mathcal{Q}_1 \subseteq \mathcal{Q}_2 \subseteq \mathcal{Q}_3$, then $d_1(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle) \leq d_1(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$ and $d_1(\langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle) \leq d_1(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$.
Similarly, we can prove that if $\mathcal{Q}_1 \subseteq \mathcal{Q}_2 \subseteq \mathcal{Q}_3$, then $d_2((\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle) \leq d_2(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$ and $d_2(\langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle) \leq d_2(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_3}, \mathbb{E}\rangle)$.

Theorem 2. Let $d(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle)$ be the distance measure between two (VSSs) $\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle$ and $\langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle$. Define $H\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle = \frac{1-d(\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle,\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle^c)}{1+d(\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle,\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle^c)}$ then $H\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle$ is an entropy of (VSSs).
Proof:

I. $H\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle = 0 \Leftrightarrow 1 - d(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle^c = 0 \Leftrightarrow d(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle^c = 1 \Leftrightarrow$ for all $\mathfrak{s} \in \mathbb{E}, \varkappa \in \mathbb{U}$, $\mathbb{T}_{\mathcal{Q}_1(\mathfrak{s})}(\varkappa) = 0, \mathbb{F}_{\mathcal{Q}_1(\mathfrak{s})}(\varkappa) = 1$ or $\mathbb{T}_{\mathcal{Q}_1(\mathfrak{s})}(\varkappa) = 1, \mathbb{F}_{\mathcal{Q}_1(\mathfrak{s})}(\varkappa) = 0$,

II. $H\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle = 1 \Leftrightarrow 1 - d(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle^c) =$
$1 + d(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle^c) \Leftrightarrow d(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, \langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle^c) = 0$
$\Leftrightarrow \langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle = \langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle^c \Leftrightarrow \mathfrak{s} \Leftrightarrow \mathbb{E}, \varkappa \in \mathbb{U}$,
$\mathbb{T}_{\mathcal{Q}_1(\mathfrak{s})}(\varkappa) = \mathbb{F}_{\mathcal{Q}_1(\mathfrak{s})}(\varkappa) = 0.5$,

III.
$$H\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle^c = \frac{1-d\left(\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle^c,\left(\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle^c\right)^c\right)}{1+d\left(\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle^c,\left(\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle^c\right)^c\right)} = \frac{1-d(\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle^c,(\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle))}{1+d(\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle^c,(\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle))} = H\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle,$$

IV. for all $\mathfrak{s} \in \mathbb{E}, \varkappa \in \mathbb{U}$ when $\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle \subseteq \langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle$, and $\mathbb{T}_{\mathcal{Q}_2(\mathfrak{s})}(\varkappa) \leq$

$\mathbb{F}_{\mathcal{Q}_2(\mathfrak{s})}(\varkappa) \implies \mathbb{T}_{\mathcal{Q}_1(\mathfrak{s})} \leq \mathbb{T}_{\mathcal{Q}_2(\mathfrak{s})} \leq \mathbb{F}_{\mathcal{Q}_2(\mathfrak{s})} \leq \mathbb{F}_{\mathcal{Q}_1(\mathfrak{s})}$; hence $|\mathbb{T}_{\mathcal{Q}_1(\mathfrak{s})}(\varkappa) - \mathbb{F}_{\mathcal{Q}_1(\mathfrak{s})}(\varkappa)| \geq |\mathbb{T}_{\mathcal{Q}_2(\mathfrak{s})}(\varkappa) - \mathbb{F}_{G(\mathfrak{s})}(\varkappa)| \implies$ $d(\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle, (\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle^c) \geq d(\langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle, (\langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle^c)$. Also $f(\varkappa) = \frac{1-\varkappa}{1+\varkappa}$ is monotone decreasing. So we have $H\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle \leq H\langle\widetilde{\mathcal{Q}_2}, \mathbb{E}\rangle$, Similarly, in the other case.

Definition 12. Let $\alpha_i^+ = (1,0)(i = 1,2,3 \ldots m)$ be the largest vague number and we call $\mathbb{A}^+ = (\alpha_1^+, \alpha_2^+, \ldots \alpha_m^+)$ As vague ideal solution,

Application 1. To obtain efficient risk management in P2P lending, certain risks are classified along with some parameters and evaluated by a team of experts. Assume that there is a set of 3 experts evaluating the five different kinds of risks with the set of parameters.

Let $\mathbb{U}$ denote the set of risks $\mathbb{U} = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5\}$ and Let $\mathbb{E}$ denote the set of parameters $\mathbb{E} = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3, \mathfrak{s}_4, \mathfrak{s}_5\}$ then the vague soft set $\langle\widetilde{\mathcal{Q}_1}, \mathbb{E}\rangle$ describes the set $\mathbb{A}$.

$\mathcal{Q}_1(\mathfrak{s}_1)(\varkappa_1) = (07 \times 10^{-1}, 01 \times 10^{-1}), \mathcal{Q}_1(\mathfrak{s}_2)(\varkappa_1) = (05 \times 10^{-1}, 05 \times 10^{-1}),$

$\mathcal{Q}_1(\mathfrak{s}_3)(\varkappa_1) = (07 \times 10^{-1}, 01 \times 10^{-1}), \mathcal{Q}_1(\mathfrak{s}_4)(\varkappa_1) = (06 \times 10^{-1}, 08 \times 10^{-1}),$

$\mathcal{Q}_1(\mathfrak{s}_5)(\varkappa_1) = (07 \times 10^{-1}, 04 \times 10^{-1}),$

$\mathcal{Q}_1(\mathfrak{s}_1)(\varkappa_2) = (08 \times 10^{-1}, 04 \times 10^{-1}), \mathcal{Q}_1(\mathfrak{s}_2)(\varkappa_2) = (07 \times 10^{-1}, 02 \times 10^{-1}),$

$\mathcal{Q}_1(\mathfrak{s}_3)(\varkappa_2) = (06 \times 10^{-1}, 09 \times 10^{-1}), \mathcal{Q}_1(\mathfrak{s}_4)(\varkappa_2) = (08 \times 10^{-1}, 09 \times 10^{-1}),$

$\mathcal{Q}_1(\mathfrak{s}_5)(\varkappa_2) = (08 \times 10^{-1}, 01 \times 10^{-1}),$

$\mathcal{Q}_1(\mathfrak{s}_1)(\varkappa_3) = (04 \times 10^{-1}, 02 \times 10^{-1}), \mathcal{Q}_1(\mathfrak{s}_2)(\varkappa_3) = (03 \times 10^{-1}, 02 \times 10^{-1}),$

$\mathcal{Q}_1(\mathfrak{s}_3)(\varkappa_3) = (02 \times 10^{-1}, 02 \times 10^{-1}), \mathcal{Q}_1(\mathfrak{s}_4)(\varkappa_3) = (04 \times 10^{-1}, 05 \times 10^{-1}),$

$\mathcal{Q}_1(\mathfrak{s}_5)(\varkappa_3) = (03 \times 10^{-1}, 07 \times 10^{-1}),$

$\mathcal{Q}_1(\mathfrak{s}_1)(\varkappa_4) = (04 \times 10^{-1}, 03 \times 10^{-1}), \mathcal{Q}_1(\mathfrak{s}_2)(\varkappa_4) = (03 \times 10^{-1}, 05 \times 10^{-1}),$
$\mathcal{Q}_1(\mathfrak{s}_3)(\varkappa_4) = (03 \times 10^{-1}, 04 \times 10^{-1}), \mathcal{Q}_1(\mathfrak{s}_4)(\varkappa_4) = (03 \times 10^{-1}, 05 \times 10^{-1}),$
$\mathcal{Q}_1(\mathfrak{s}_5)(\varkappa_4) = (04 \times 10^{-1}, 03 \times 10^{-1}),$
$\mathcal{Q}_1(\mathfrak{s}_1)(\varkappa_5) = (03 \times 10^{-1}, 07 \times 10^{-1}), \mathcal{Q}_1(\mathfrak{s}_2)(\varkappa_5) = (02 \times 10^{-1}, 01 \times 10^{-1}),$
$\mathcal{Q}_1(\mathfrak{s}_3)(\varkappa_5) = (03 \times 10^{-1}, 04 \times 10^{-1}), \mathcal{Q}_1(\mathfrak{s}_4)(\varkappa_5) = (03 \times 10^{-1}, 09 \times 10^{-1}),$
$\mathcal{Q}_1(\mathfrak{s}_5)(\varkappa_5) = (04 \times 10^{-1}, 01 \times 10^{-1}).$

The vague soft set $\langle \widetilde{\mathcal{Q}_2}, \mathbb{E} \rangle$ describes the set $\mathbb{B}$
$\mathcal{Q}_2(\mathfrak{s}_1)(\varkappa_1) = (04 \times 10^{-1}, 02 \times 10^{-1}), \mathcal{Q}_2(\mathfrak{s}_2)(\varkappa_1) = (03 \times 10^{-1}, 01 \times 10^{-1}),$
$\mathcal{Q}_2(\mathfrak{s}_3)(\varkappa_1) = (05 \times 10^{-1}, 09 \times 10^{-1}), \mathcal{Q}_2(\mathfrak{s}_4)(\varkappa_1) = (04 \times 10^{-1}, 04 \times 10^{-1}),$
$\mathcal{Q}_2(\mathfrak{s}_5)(\varkappa_1) = (04 \times 10^{-1}, 01 \times 10^{-1}),$
$\mathcal{Q}_2(\mathfrak{s}_1)(\varkappa_2) = (03 \times 10^{-1}, 05 \times 10^{-1}), \mathcal{Q}_2(\mathfrak{s}_2)(\varkappa_2) = (03 \times 10^{-1}, 02 \times 10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_3)(\varkappa_2) = (03 \times 10^{-1}, 07 \times 10^{-1}), \mathcal{Q}_2(\mathfrak{s}_4)(\varkappa_2) = (04 \times 10^{-1}, 01 \times 10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_5)(\varkappa_2) = (03 \times 10^{-1}, 01 \times 10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_1)(\varkappa_3) = (04 \times 10^{-1}, 05 \times 10^{-1}), \mathcal{Q}_2(\mathfrak{s}_2)(\varkappa_3) = (03 \times 10^{-1}, 08 \times 10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_3)(\varkappa_3) = (03 \times 10^{-1}, 01 \times 10^{-1}), \mathcal{Q}_2(\mathfrak{s}_4)(\varkappa_3) = (04 \times 10^{-1}, 08 \times 10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_5)(\varkappa_3) = (02 \times 10^{-1}, 06 \times 10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_1)(\varkappa_4) = (03 \times 10^{-1}, 01 \times 10^{-1}), \mathcal{Q}_2(\mathfrak{s}_2)(\varkappa_4) = (03 \times 10^{-1}, 02 \times 10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_3)(\varkappa_4) = (03 \times 10^{-1}, 05 \times 10^{-1}), \mathcal{Q}_2(\mathfrak{s}_4)(\varkappa_4) = (05 \times 10^{-1}, 04 \times 10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_5)(\varkappa_4) = (04 \times 10^{-1}, 08 \times 10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_1)(\varkappa_5) = (04 \times 10^{-1}, 08 \times 10^{-1}), \mathcal{Q}_2(\mathfrak{s}_2)(\varkappa_5) = (03 \times 10^{-1}, 08 \times 10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_3)(\varkappa_5) = (04 \times 10^{-1}, 06 \times 10^{-1}), \mathcal{Q}_2(\mathfrak{s}_4)(\varkappa_5) = (04 \times 10^{-1}, 01 \times 10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_5)(\varkappa_5) = (03 \times 10^{-1}, 08 \times 10^{-1}),$

The vague soft set

$\mathcal{Q}_3(\mathfrak{s}_1)(\varkappa_1) = (04 \times 10^{-1}, 01 \times 10^{-1}), \mathcal{Q}_3(\mathfrak{s}_2)(\varkappa_1) = (03 \times 10^{-1}, 01 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_3)(\varkappa_1) = (03 \times 10^{-1}, 03 \times 10^{-1}), \mathcal{Q}_3(\mathfrak{s}_4)(\varkappa_1) = (03 \times 10^{-1}, 07 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_5)(\varkappa_1) = (05 \times 10^{-1}, 01 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_1)(\varkappa_2) = (03 \times 10^{-1}, 05 \times 10^{-1}), \mathcal{Q}_3(\mathfrak{s}_2)(\varkappa_2) = (02 \times 10^{-1}, 03 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_3)(\varkappa_2) = (03 \times 10^{-1}, 01 \times 10^{-1}), \mathcal{Q}_3(\mathfrak{s}_4)(\varkappa_2) = (03 \times 10^{-1}, 06 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_5)(\varkappa_2) = (05 \times 10^{-1}, 01 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_1)(\varkappa_3) = (03 \times 10^{-1}, 09 \times 10^{-1}), \mathcal{Q}_3(\mathfrak{s}_2)(\varkappa_3) = (03 \times 10^{-1}, 05 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_3)(\varkappa_3) = (03 \times 10^{-1}, 04 \times 10^{-1}), \mathcal{Q}_3(\mathfrak{s}_4)(\varkappa_3) = (03 \times 10^{-1}, 08 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_5)(\varkappa_3) = (03 \times 10^{-1}, 07 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_1)(\varkappa_4) = (03 \times 10^{-1}, 06 \times 10^{-1}), \mathcal{Q}_3(\mathfrak{s}_2)(\varkappa_4) = (03 \times 10^{-1}, 02 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_3)(\varkappa_4) = (03 \times 10^{-1}, 06 \times 10^{-1}), \mathcal{Q}_3(\mathfrak{s}_4)(\varkappa_4) = (03 \times 10^{-1}, 09 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_5)(\varkappa_4) = (03 \times 10^{-1}, 08 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_1)(\varkappa_5) = (04 \times 10^{-1}, 09 \times 10^{-1}), \mathcal{Q}_3(\mathfrak{s}_2)(\varkappa_5) = (04 \times 10^{-1}, 09 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_3)(\varkappa_5) = (04 \times 10^{-1}, 07 \times 10^{-1}), \mathcal{Q}_3(\mathfrak{s}_4)(\varkappa_5) = (03 \times 10^{-1}, 08 \times 10^{-1}),$
$\mathcal{Q}_3(\mathfrak{s}_5)(\varkappa_5) = (03 \times 10^{-1}, 06 \times 10^{-1}),$

$\langle \widetilde{\mathcal{Q}_3}, \mathbb{E} \rangle$ describes the set $\mathbb{C}$.

So we have

$H_1\langle\widetilde{\mathcal{Q}_2},\mathbb{E}\rangle = 01068\times10^{-4}, H_2\langle\widetilde{\mathcal{Q}_2},\mathbb{E}\rangle = 00645\times10^{-4}, H_3\langle\widetilde{\mathcal{Q}_2},\mathbb{E}\rangle = 00922\times10^{-4}$

$H_4\langle\widetilde{\mathcal{Q}_2},\mathbb{E}\rangle = 00917\times10^{-4}, H_5\langle\widetilde{\mathcal{Q}_2},\mathbb{E}\rangle = 00856\times10^{-4},$

$H_1\langle\widetilde{\mathcal{Q}_3},\mathbb{E}\rangle = 01032\times10^{-4}, H_2\langle\widetilde{\mathcal{Q}_3},\mathbb{E}\rangle = 00743\times10^{-4}, H_3\langle\widetilde{\mathcal{Q}_3},\mathbb{E}\rangle = 01222\times10^{-4}$

$H_4\langle\widetilde{\mathcal{Q}_3},\mathbb{E}\rangle = 00823\times10^{-4}, H_5\langle\widetilde{\mathcal{Q}_3},\mathbb{E}\rangle = 00707\times10^{-4},$

Therefore, $H\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle = 00986\times10^{-4}, H\langle\widetilde{\mathcal{Q}_2},\mathbb{E}\rangle = 00884\times10^{-4}, H\langle\widetilde{\mathcal{Q}_3},\mathbb{E}\rangle = 00906\times10^{-4}$.

From the computation we have $H\langle\widetilde{\mathcal{Q}_2},\mathbb{E}\rangle \le H\langle\widetilde{\mathcal{Q}_3},\mathbb{E}\rangle \le H\langle\widetilde{\mathcal{Q}_1},\mathbb{E}\rangle$. Therefore, the set $\mathbb{B}$ has larger possibility than the set $\mathbb{A}$ and $\mathbb{C}$.

Example 1. Let there six sets $\mathbb{A}_1,\mathbb{A}_2,\mathbb{A}_3,\mathbb{A}_4,\mathbb{A}_5,\mathbb{A}_6$ to be determined and let $U=\{\varkappa_1\}$ and E denotes the set of parameters $\mathbb{E}=\{\mathfrak{s}_1,\mathfrak{s}_2,\mathfrak{s}_{1_3}\}$ and the vague soft sets are given as $\langle\widetilde{\mathcal{Q}_1},\mathbb{A}_1\rangle,\langle\widetilde{\mathcal{Q}_2},\mathbb{A}_2\rangle,\dots\dots\langle\widetilde{\mathcal{Q}_6},\mathbb{A}_6\rangle$.

$\mathcal{Q}_1(\mathfrak{s}_1)(\varkappa_1) = (04\times10^{-1}, 03\times10^{-1}), \mathcal{Q}_1(\mathfrak{s}_2)(\varkappa_1) = (06\times10^{-1}, 01\times10^{-1}),$

$\mathcal{Q}_1(\mathfrak{s}_3)(\varkappa_1) = (05\times10^{-1}, 04\times10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_1)(\varkappa_1) = (05\times10^{-1}, 02\times10^{-1}), \mathcal{Q}_2(\mathfrak{s}_2)(\varkappa_1) = (03\times10^{-1}, 04\times10^{-1}),$

$\mathcal{Q}_2(\mathfrak{s}_3)(\varkappa_1) = (08\times10^{-1}, 01\times10^{-1}),$

$\mathcal{Q}_3(\mathfrak{s}_1)(\varkappa_1) = (07\times10^{-1}, 02\times10^{-1}), \mathcal{Q}_3(\mathfrak{s}_2)(\varkappa_1) = (03\times10^{-1}, 07\times10^{-1}),$

$\mathcal{Q}_3(\mathfrak{s}_3)(\varkappa_1) = (06\times10^{-1}, 02\times10^{-1}),$

$\mathcal{Q}_4(\mathfrak{s}_1)(\varkappa_1) = (04\times10^{-1}, 03\times10^{-1}), \mathcal{Q}_4(\mathfrak{s}_2)(\varkappa_1) = (06\times10^{-1}, 02\times10^{-1}),$

$\mathcal{Q}_4(\mathfrak{s}_3)(\varkappa_1) = (07\times10^{-1}, 01\times10^{-1}).$

$\mathcal{Q}_5(\mathfrak{s}_1)(\varkappa_1) = (06\times10^{-1}, 02\times10^{-1}), \mathcal{Q}_5(\mathfrak{s}_2)(\varkappa_1) = (05\times10^{-1}, 01\times10^{-1}),$

$\mathcal{Q}_5(\mathfrak{s}_3)(\varkappa_1) = (04\times10^{-1}, 06\times10^{-1}),$

$\mathcal{Q}_6(\mathfrak{s}_1)(\varkappa_1) = (06\times10^{-1}, 03\times10^{-1}), \mathcal{Q}_6(\mathfrak{s}_2)(\varkappa_1) = (07\times10^{-1}, 02\times10^{-1}),$

$\mathcal{Q}_6(\mathfrak{s}_3)(\varkappa_1) = (05\times10^{-1}, 04\times10^{-1}).$

The vague soft ideal solution $\langle\mathcal{Q},\mathbb{A}^+\rangle$ is given by $\mathcal{Q}(\mathfrak{s}_1)(\varkappa_1) = (1,0), \mathcal{Q}(\mathfrak{s}_2)(\varkappa_1) = (1,0), \mathcal{Q}(\mathfrak{s}_3)(\varkappa_1) = (1,0),$

we have

$d_1(\langle\widetilde{\mathcal{Q}_1},\mathbb{A}_1\rangle,\langle\mathcal{Q},\mathbb{A}^+\rangle) = 05664\times10^{-4}, d_1(\langle\widetilde{\mathcal{Q}_2},\mathbb{A}_2\rangle,\langle\mathcal{Q},\mathbb{A}^+\rangle) = 06\times10^{-1},$

$d_1(\langle\widetilde{\mathcal{Q}_3},\mathbb{A}_3\rangle,\langle\mathcal{Q},\mathbb{A}^+\rangle) = 07331\times10^{-4}, d_1(\langle\widetilde{\mathcal{Q}_4},\mathbb{A}_4\rangle,\langle\mathcal{Q},\mathbb{A}^+\rangle) = 0.7,$

$d_1(\langle\widetilde{\mathcal{Q}_5},\mathbb{A}_5\rangle,\langle\mathcal{Q},\mathbb{A}^+\rangle) = 05\times10^{-1}, d_1(\langle\widetilde{\mathcal{Q}_6},\mathbb{A}_6\rangle,\langle\mathcal{Q},\mathbb{A}^+\rangle) = 06331\times10^{-4},$

Since,

$d_1(\langle\widetilde{\mathcal{Q}_5},\mathbb{A}_5\rangle,\langle\mathcal{Q},\mathbb{A}^+\rangle) < d_1(\langle\widetilde{\mathcal{Q}_1},\mathbb{A}_1\rangle,\langle\mathcal{Q},\mathbb{A}^+\rangle) < d_1(\langle\widetilde{\mathcal{Q}_4},\mathbb{A}_4\rangle,\langle\mathcal{Q},\mathbb{A}^+\rangle) <$

$d_1(\langle\widetilde{\mathcal{Q}_6},\mathbb{A}_6\rangle,\langle\mathcal{Q},\mathbb{A}^+\rangle) < d_1(\langle\widetilde{\mathcal{Q}_2},\mathbb{A}_2\rangle,\langle\mathcal{Q},\mathbb{A}^+\rangle) < d_1(\langle\widetilde{\mathcal{Q}_3},\mathbb{A}_3\rangle,\langle\mathcal{Q},\mathbb{A}^+\rangle)$ then $\langle\widetilde{\mathcal{Q}_5},\mathbb{A}_5\rangle > \langle\widetilde{\mathcal{Q}_1},\mathbb{A}_1\rangle > \langle\widetilde{\mathcal{Q}_4},\mathbb{A}_4\rangle > \langle\widetilde{\mathcal{Q}_6},\mathbb{A}_6\rangle > \langle\widetilde{\mathcal{Q}_2},\mathbb{A}_2\rangle > \langle\widetilde{\mathcal{Q}_1},\mathbb{A}_1\rangle$. Hence the most desirable candidate is $\mathbb{A}_5$.

Definition 15. Let $\mathbb{U}$ be a universe of discourse, an unterval valued vague set (IVVS) $\mathcal{S}$ in $\mathbb{U}$ is characterized by $\mathbb{T}_{\mathcal{S}}(\varkappa)$ and $F_{\mathcal{S}}(\varkappa)$. For each point $\varkappa$ in $\mathbb{U}$, we have that $\mathbb{T}_{\mathcal{S}}(\varkappa), \mathbb{F}_{\mathcal{S}}(\varkappa) \in [0,1]$.

For two IVVS, $\mathcal{S}_{1_{IVVS}} = \{<\varkappa, [\mathbb{T}^{\mathcal{L}}_{\mathcal{S}_1}(\varkappa), \mathbb{T}^{\mathcal{U}}_{\mathcal{S}_1}(\varkappa)], [\mathbb{F}^{\mathcal{L}}_{\mathcal{S}_1}(\varkappa), \mathbb{F}^{\mathcal{U}}_{\mathcal{S}_1}(\varkappa)] >| \varkappa \in \mathbb{U}\}$
and $\mathcal{S}_{2_{IVVS}} = \{<\varkappa, [\mathbb{T}^{\mathcal{L}}_{\mathcal{S}_2}(\varkappa), \mathbb{T}^{\mathcal{U}}_{\mathcal{S}_2}(\varkappa)], [\mathbb{F}^{\mathcal{L}}_{\mathcal{S}_2}(\varkappa), \mathbb{F}^{\mathcal{U}}_{\mathcal{S}_2}(\varkappa)] >| \varkappa \in \mathbb{U}\}$ the two relations are defined as follows:

I. $\mathcal{S}_{1_{IVVS}} \subseteq \mathcal{S}_{2_{IVNS}}$s if and only if $\mathbb{T}^{L}_{\mathcal{S}_1}(\varkappa) \le \mathbb{T}^{\mathcal{L}}_{\mathcal{S}_2}(\varkappa), \mathbb{T}^{\mathcal{U}}_{\mathcal{S}_1}(\varkappa) \le \mathbb{T}^{\mathcal{U}}_{\mathcal{S}_2}(\varkappa), \mathbb{F}^{\mathcal{L}}_{\mathcal{S}_1}(\varkappa) \ge \mathbb{F}^{\mathcal{L}}_{\mathcal{S}_2}(\varkappa), \mathbb{F}^{\mathcal{U}}_{\mathcal{S}_1}(\varkappa) \ge F^{\mathcal{U}}_{\mathcal{S}_2}(\varkappa),$

II. $\mathcal{S}_{1_{IVVS}} = \mathcal{S}_{2_{IVVS}}$ if and only if, $\mathbb{T}^{\mathcal{L}}_{\mathcal{S}_1}(\varkappa_i) = \mathbb{T}^{\mathcal{L}}_{\mathcal{S}_2}(\varkappa_i), \mathbb{T}^{\mathcal{U}}_{\mathcal{S}_1}(\varkappa_i) = \mathbb{T}^{\mathcal{U}}_{\mathcal{S}_2}(\varkappa_i)$, $\mathbb{F}^{\mathcal{L}}_{\mathcal{S}_1}(\varkappa_i) = \mathbb{F}^{\mathcal{L}}_{\mathcal{S}_2}(\varkappa_i)$ and $F^{\mathcal{U}}_{\mathcal{S}_1}(\varkappa_i) = F^{\mathcal{U}}_{\mathcal{S}_2}(\varkappa_i)$ for any $\varkappa \in \mathbb{U}$.

The complement of $\mathcal{S}_{1_{IVVS}}$ is denoted by $\mathcal{S}_{1_{IVVS}}^{o}$ and is defined by
$\mathcal{S}_{1_{IVNS}}^{o} = \{<\varkappa, [\mathbb{F}^{\mathcal{L}}_{\mathcal{S}_1}(\varkappa), \mathbb{T}^{\mathcal{U}}_{\mathcal{S}_1}(\varkappa)] >, [\mathbb{T}^{\mathcal{L}}_{\mathcal{S}_1}(\varkappa), \mathbb{T}^{\mathcal{U}}_{\mathcal{S}_1}(\varkappa)] : \varkappa \in \mathbb{U}\}$.

Definition 16. Let $\mathbb{U}$ be an universe, IVV ($\mathbb{U}$) denotes the set of all (IVVSs) of $\mathbb{U}$ and $\mathbb{E}$ be a set of parameters. The collection $(\mathcal{S}_1,\mathbb{E})$ is termed to be (IVVSSs) over $\mathbb{U}$ denoted by $\widetilde{\mathbb{I}_1}$, where $\mathcal{S}_1$ is a mapping given by $\mathcal{S}_1: \mathbb{E} \to IVV(\mathbb{U})$. It can be written as $\widetilde{\mathbb{I}_1} = \{(\varkappa, \mathcal{S}_1(\varkappa)) : \varkappa \in \mathbb{E}\}$.

Here, $\mathcal{S}_1$ which is (IVVSSs), is called approximate function of (IVVSSs) $\widetilde{\mathbb{I}_1}$ and $\mathcal{S}_1(\varkappa)$ is called $\varkappa$- approximate value of $\varkappa \in \mathbb{E}$.

Generally, $\mathcal{S}_1,\mathcal{S}_2,\mathcal{S}_3,\dots$ will be used as an approximate functions of $\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2},\widetilde{\mathbb{I}_3}\dots$ respectively and the sets of all

(IVVSSs), over $\mathbb{U}$ will be denoted by $\mathrm{IVVS}(\mathbb{U})$.

Then a relation form of $\widetilde{\mathbb{I}_1}$ is defined by $\mathrm{R}_{\mathcal{S}_1} = \{(\mathrm{r}_{\mathcal{S}_1}(\mathfrak{s}, \varkappa)/(\mathfrak{s}, \varkappa)) : \varkappa \in \mathbb{U}, \mathfrak{s} \in \mathbb{E}\}$

Where $\mathrm{r}_{\mathcal{S}_1} : \mathrm{E x \mathbb{U}} \rightarrow \mathrm{IVVS}(\mathbb{U})$ and $\mathrm{r}_{\mathcal{S}_1}(\mathfrak{s}_i, \varkappa_j) = \mathrm{a}_{ij}$ for all $\mathfrak{s}_i \in \mathbb{E}$ and $\varkappa_j \in \mathbb{U}$.

Here,

I. $\widetilde{\mathbb{I}_1}$ is an (IVVSS) of $\widetilde{\mathbb{I}_2}$, denoted by $\widetilde{\mathbb{I}_1} \Subset \widetilde{\mathbb{I}_2}$, if $\mathcal{S}_1(\mathfrak{s}) \subseteq \mathcal{S}_2(\mathfrak{s})$ for all $\mathfrak{s} \in \mathbb{E}$.

II. $\widetilde{\mathbb{I}_1}$ is an (IVVS) equals to $\widetilde{\mathbb{I}_2}$, denoted by $\widetilde{\mathbb{I}_1} = \widetilde{\mathbb{I}_2}$, if $\mathcal{S}_1(\mathfrak{s}) = \mathcal{S}_2(\mathfrak{s})$ for all $\mathfrak{s} \in \mathbb{E}$.

III. The complement of $\widetilde{\mathbb{I}_1}$ is denoted by $\widetilde{\mathbb{I}_1}^{c}$, and is defined by $\widetilde{\mathbb{I}_1}^{c} = \{(\varkappa, \mathcal{S}_1{}^{o}(\varkappa)) : \mathfrak{s} \in \mathbb{E}\}$.

Example 2. Suppose that $\mathbb{U}$ is set of houses under consideration, say $\mathbb{U} = \{\varkappa_1, \varkappa_2, \varkappa_3\}$. Let $\mathbb{E}$ be the set of some attributes of such houses, say $\mathbb{E} = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3, \mathfrak{s}_4\}$, where $\mathfrak{s}_1, \mathfrak{s}_2, \dots, \mathfrak{s}_4$ stand for the attributes "expensive","beautiful", "wooden","cheap","modern", and "in bad repair “ respectively.

In this case we give an (IVVSS) as;

$$\widetilde{\mathbb{I}_1} = \begin{bmatrix} \left(\left(\mathfrak{s}_1, \begin{pmatrix} < \varkappa_1, [05 \times 10^{-1}, 06 \times 10^{-1}], [03 \times 10^{-1}, 04 \times 10^{-1}] >, \\ < \varkappa_2, [05 \times 10^{-1}, 06 \times 10^{-1}], [03 \times 10^{-1}, 04 \times 10^{-1}] >, \\ < \varkappa_3, [05 \times 10^{-1}, 06 \times 10^{-1}], [03 \times 10^{-1}, 04 \times 10^{-1}] >, \end{pmatrix}\right)\right), \\ \left(\left(\mathfrak{s}_2, \begin{pmatrix} < \varkappa_1, [02 \times 10^{-1}, 03 \times 10^{-1}], [03 \times 10^{-1}, 05 \times 10^{-1}] >, \\ < \varkappa_2, [04 \times 10^{-1}, 06 \times 10^{-1}], [02 \times 10^{-1}, 03 \times 10^{-1}] >, \\ < \varkappa_3, [05 \times 10^{-1}, 06 \times 10^{-1}], [03 \times 10^{-1}, 04 \times 10^{-1}] >, \end{pmatrix}\right)\right), \\ \left(\left(\mathfrak{s}_3, \begin{pmatrix} < \varkappa_1, [03 \times 10^{-1}, 05 \times 10^{-1}], [02 \times 10^{-1}, 04 \times 10^{-1}] >, \\ < \varkappa_2, [02 \times 10^{-1}, 05 \times 10^{-1}], [04 \times 10^{-1}, 05 \times 10^{-1}] >, \\ < \varkappa_3, [05 \times 10^{-1}, 06 \times 10^{-1}], [03 \times 10^{-1}, 04 \times 10^{-1}] >, \end{pmatrix}\right)\right), \\ \left(\left(\mathfrak{s}_4, \begin{pmatrix} < \varkappa_1, [04 \times 10^{-1}, 06 \times 10^{-1}], [03 \times 10^{-1}, 04 \times 10^{-1}] >, \\ < \varkappa_2, [04 \times 10^{-1}, 06 \times 10^{-1}], [02 \times 10^{-1}, 03 \times 10^{-1}] >, \\ < \varkappa_3, [03 \times 10^{-1}, 04 \times 10^{-1}], [01 \times 10^{-1}, 04 \times 10^{-1}] >, \end{pmatrix}\right)\right) \end{bmatrix}.$$

Definition 18. Let $\mathbb{E}$ be a set of parameters. Suppose that $\widetilde{\mathbb{I}_1} = \langle \mathcal{S}_1, \mathbb{E} \rangle, \widetilde{\mathbb{I}_2} = \langle \mathcal{S}_2, \mathbb{E} \rangle$ and $\widetilde{\mathbb{I}_3} = \langle \mathcal{S}_3, \mathbb{E} \rangle$ be three (IVVSSs) in universe U. Assume d is a mapping.

d : IVVS $(\mathbb{U}) \times$ IVVS$(\mathbb{U}) \rightarrow [0,1]$. If d satisfies the following properties $((1) - (4))$:

$$\mathrm{d}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) \geq 0, \tag{1}$$

$$\mathrm{d}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = \mathrm{d}(\widetilde{\mathbb{I}_2}, \widetilde{\mathbb{I}_1}), \tag{2}$$

$$\mathrm{d}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = 0 \ \text{ if } \ \widetilde{\mathbb{I}_2} = \widetilde{\mathbb{I}_1}, \tag{3}$$

$$\mathrm{d}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) + \mathrm{d}(\widetilde{\mathbb{I}_2}, \widetilde{\mathbb{I}_3}) \geq \mathrm{d}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_3}) . \tag{4}$$

Hence d($\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}$) is called distance measure between (IVVSSs) $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$.

Definition 19. A real function $\mathrm{S} : \mathrm{IVS}(\mathbb{U}) \times \mathrm{INS}(\mathbb{U}) \rightarrow [0,1]$ is named a similarity measure between two (IVVSSs) $\widetilde{\mathbb{I}_1} = (\mathcal{S}_1, \mathbb{E})$ and $\widetilde{\mathbb{I}_2} = (\mathcal{S}_2, \mathbb{E})$ if S satisfies all the following properties:

$$\mathcal{S}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) \in [0,1], \tag{1}$$

$$\mathrm{S}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_1}) = \mathrm{S}(\widetilde{\mathbb{I}_2}, \widetilde{\mathbb{I}_2}) = 1, \tag{2}$$

$$\mathrm{S}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = \mathrm{S}(\widetilde{\mathbb{I}_2}, \widetilde{\mathbb{I}_1}), \tag{3}$$

$$\mathrm{S}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_3}) \leq \mathrm{S}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) \ \text{ and } \ \mathrm{S}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_3}) \leq \mathrm{S}(\widetilde{\mathbb{I}_2}, \widetilde{\mathbb{I}_3}) \ \text{ if } \ \widetilde{\mathbb{I}_1} \subseteq \widetilde{\mathbb{I}_2} \subseteq \widetilde{\mathbb{I}_3}. \tag{4}$$

Hence $\mathrm{S}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2})$ is called a similarity measure between (IVVSSs) $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$.

3 | Distance Measure for Interval Valued Vague Soft Sets

The definitions of the Hamming and Euclidean distances between (IVVSSs) and the similarity measures based on the distances are presented in this section.

Definition 20. Hamming distance between two (IVVS)is given as follow:

$D(\mathcal{S}_1,\mathcal{S}_2)=\frac{1}{6}\sum_{i=1}^{n}\Big[\big|\ \mathbb{T}\mathcal{j}_{\mathcal{S}_1}^{\mathcal{L}}(\varkappa_i)-\mathbb{T}_{\mathcal{S}_2}^{\mathcal{L}}(\varkappa_i)\big|+\big|\mathbb{T}_{\mathcal{S}_1}^{\mathcal{U}}(\varkappa_i)-\mathbb{T}_{\mathcal{S}_2}^{\mathcal{U}}(\varkappa_i)\big|$
$+|\mathbb{F}_{\mathcal{S}_1}^{\mathcal{L}}(\varkappa_i)-\mathbb{F}_{\mathcal{S}_2}^{\mathcal{L}}(\varkappa_i)|+|\mathbb{F}_{\mathcal{S}_1}^{\mathcal{L}}(\varkappa_i)-\mathbb{F}(\varkappa_i)\ |\Big].$

We extended it to the case of (IVVSSs) as follows:

Definition 21. Let $\widetilde{\mathbb{I}_1}=(\widetilde{\mathcal{S}_1},\mathbb{E})=[a_{ij}]_{mxn}$ and $\widetilde{\mathbb{I}_2}=(\widetilde{\mathcal{S}_2},\mathbb{E})=[b_{ij}]_{mxn}$ be two (IVVSSs).

$$\widetilde{\mathcal{S}_1}(\mathfrak{s})=\left\{<\varkappa,\left[\mathbb{T}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}^{\mathcal{L}}(\varkappa),\mathbb{T}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}^{\mathcal{U}}(\varkappa)\right],\left[\mathbb{F}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}^{\mathcal{L}}(\varkappa),\mathbb{T}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}^{\mathcal{U}}(\varkappa)\right]>:\varkappa\in\mathbb{U}\right\}$$
$$\widetilde{\mathcal{S}_2}(\mathfrak{s})=\left\{<\varkappa,\left[\mathbb{T}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}^{\mathcal{L}}(\varkappa),\mathbb{T}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}^{\mathcal{U}}(\varkappa)\right],\left[\mathbb{F}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}^{\mathcal{L}}(\varkappa),\mathbb{F}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}^{\mathcal{U}}(\varkappa)\right]>:\varkappa\in\mathbb{U}\right\}'$$

Then we define the following distances for $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$

The Hamming distance $d_{IVVSS}^{H}\ (\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})$, (1)

Where $\Delta_{ij}^{\mathcal{L}}\mathbb{T}=\mathbb{T}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}^{\mathcal{L}}(\varkappa_i)-\mathbb{T}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}^{\mathcal{L}}(\varkappa_i),\Delta_{ij}^{\mathcal{U}}\mathbb{T}=\mathbb{T}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}^{\mathcal{U}}(\varkappa_i)-\mathbb{T}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}^{\mathcal{U}}(\varkappa_i)$,
$,\Delta_{ij}^{\mathcal{L}}\mathbb{F}=\mathbb{F}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}^{\mathcal{L}}(\varkappa_i)-\mathbb{F}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}^{\mathcal{L}}(\varkappa_i)$ and $\Delta_{ij}^{\mathcal{U}}\mathbb{F}=\mathbb{F}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}^{\mathcal{U}}(\varkappa_i)-\mathbb{F}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}^{\mathcal{U}}(\varkappa_i)$.

The normalized Hamming distance $d_{IVVSS}^{nH}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})$, (2)

$$d_{IVVSS}^{nH}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\frac{d_{IVNSS}^{H}\ (\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})}{mn}.$$

The Euclidean distance $d_{IVNSS}^{E}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})$, (3)

$$d_{IVVSS}^{E}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{\left(\Delta_{ij}^{\mathcal{L}}\mathbb{T}^2)+(\Delta_{ij}^{\mathcal{U}}\mathbb{T}\right)^2+\left((\Delta_{i}^{\mathcal{L}}\mathbb{F})^2+(\Delta_{ij}^{\mathcal{U}}\mathbb{F})^2\right)}{6}},$$

Where $\Delta_{ij}^{\mathcal{L}}\mathbb{T}=\mathbb{T}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}^{\mathcal{L}}(\varkappa_i)-\mathbb{T}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}^{\mathcal{L}}(\varkappa_i),\Delta_{ij}^{\mathcal{U}}\mathbb{T}=\mathrm{T}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}^{\mathcal{U}}(\varkappa_i)-\mathbb{T}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}^{\mathcal{U}}(\varkappa_i),\Delta_{ij}^{\mathcal{L}}\mathbb{F}=\mathbb{F}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}^{\mathcal{L}}(\varkappa_i)-\mathbb{F}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}^{\mathcal{L}}(\varkappa_i)$ and $\Delta_{ij}^{\mathcal{U}}\mathbb{F}=\mathbb{F}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}^{\mathcal{U}}(\varkappa_i)-\mathbb{F}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}^{\mathcal{U}}(\varkappa_i)$.

The normalized Euclidean distance $d_{IVVSS}^{nE}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})$, (4)

$$d_{IVVSS}^{n\mathbb{E}}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\frac{d_{IVNSS}^{\mathbb{E}}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})}{\sqrt{mn}}.$$

Here,it is clear that the following properties hold:

$0\le d_{IVVSS}^{H}\ (\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})\le mn$ and $0\le d_{IVVSS}^{nH}\ (\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})\le 1$; (1)

$0\le d_{IVVSS}^{\mathbb{E}}\ (\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})\le\sqrt{mn}$ and $0\le d_{IVVSS}^{n\mathbb{E}}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})\le 1$; (2)

Example 3. Assume that two (IVVSSs) $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$ are defined as follows

$$\widetilde{\mathcal{S}_1}(\mathfrak{s}_1)=\begin{pmatrix}<\varkappa_1,[05\times10^{-1},06\times10^{-1}],[03\times10^{-1},04\times10^{-1}]>,\\<\varkappa_2,[05\times10^{-1},06\times10^{-1}],[03\times10^{-1},04\times10^{-1}]>\end{pmatrix},$$
$$\widetilde{\mathcal{S}_1}(\mathfrak{s}_2)=\begin{pmatrix}<\varkappa_1,[02\times10^{-1},03\times10^{-1}],[03\times10^{-1},06\times10^{-1}]>,\\<\varkappa_2,[04\times10^{-1},06\times10^{-1}],[02\times10^{-1},03\times10^{-1}]>\end{pmatrix}$$
$$\widetilde{\mathcal{S}_2}(\mathfrak{s}_1)=\begin{pmatrix}<\varkappa_1,[03\times10^{-1},04\times10^{-1}],[02\times10^{-1},04\times10^{-1}]>,\\<\varkappa_2,[02\times10^{-1},05\times10^{-1}],[04\times10^{-1},05\times10^{-1}]>\end{pmatrix},$$
$$\widetilde{\mathcal{S}_2}(\mathfrak{s}_2)=\begin{pmatrix}<\varkappa_1,[04\times10^{-1},06\times10^{-1}],[03\times10^{-1},04\times10^{-1}]>,\\<\varkappa_2,[03\times10^{-1},04\times10^{-1}],[01\times10^{-1},04\times10^{-1}]>\end{pmatrix},$$

$$d_{IVVSS}^{H}\ (\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\sum_{j=1}^{2}\sum_{i=1}^{2}\frac{[|\Delta_{ij}^{\mathcal{L}}\mathbb{T}|+|\Delta_{ij}^{\mathcal{U}}\mathbb{T}|+|\Delta_{ij}^{\mathcal{L}}\mathbb{F}|+|\Delta_{ij}^{\mathcal{U}}\mathbb{F}|]}{6},$$
$$=\frac{(|05-03|+|06-04|+|03-02|+|04-04|)\times10^{-1}}{6}$$
$$+\frac{(|05-02|+|06-05|+|03-04|+|04-05|)\times10^{-1}}{6}+$$
$$\frac{(|02-04|+|03-06|+|03-03|+|06-04|)\times10^{-1}}{6}+$$
$$\frac{(|04-03|+|06-04|+|02-01|+|03-04|)\times10^{-1}}{6},$$

$d_{IVVSS}^{H}\ (\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=0.71.$

Definition 22. Generalized weighted distance measure between two (IVVSs). Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two (IVVSs), then a generalized interval valued vague weighted distance measure (IVVWDM) between $\mathcal{S}_1$ and $\mathcal{S}_2$ as follows:

$$d_\gamma(\mathcal{S}_1,\mathcal{S}_2)=\left\{\frac{1}{6}\sum_{j=1}^{m}\sum_{i=1}^{n} w_i\left[\left|\mathbb{T}^{\mathcal{L}}_{\mathcal{S}_1}(\varkappa_i)-\mathbb{T}^{\mathcal{L}}_{\mathcal{S}_2}(\varkappa_i)\right|^\gamma+\left|\mathbb{T}^{\mathcal{U}}_{\mathcal{S}_1}(\varkappa_i)-\mathbb{T}^{\mathcal{U}}_{\mathcal{S}_2}(\varkappa_i)\right|^\gamma+\left|\mathbb{F}^{\mathcal{L}}_{\mathcal{S}_1}(\varkappa_i)-\mathbb{F}^{\mathcal{L}}_{\mathcal{S}_2}(\varkappa_i)\right|^\gamma+\left|\mathbb{F}^{\mathcal{U}}_{\mathcal{S}_1}(\varkappa_i)-\mathbb{F}^{\mathcal{U}}_{\mathcal{S}_2}(\varkappa_i)\right|^\gamma\right]\right\}^{\frac{1}{\gamma}}.$$

where

$\gamma>0$ and $\mathbb{T}^{\mathcal{L}}_{\mathcal{S}_1}(\varkappa_i),\mathbb{T}^{\mathcal{U}}_{\mathcal{S}_1}(\varkappa_i),\mathbb{F}^{\mathcal{L}}_{\mathcal{S}_1}(\varkappa_i),\mathbb{F}^{\mathcal{U}}_{\mathcal{S}_1}(\varkappa_i),\mathbb{T}^{\mathcal{L}}_{\mathcal{S}_2}(\varkappa_i),\mathbb{T}^{\mathcal{U}}_{\mathcal{S}_2}(\varkappa_i),\mathbb{F}^{\mathcal{L}}_{\mathcal{S}_2}(\varkappa_i),\mathbb{F}^{\mathcal{U}}_{\mathcal{S}_2}(\varkappa_i),\in[0,1]$.

we extended the above equation distance to the case of (IVVSs) between $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$ as follow:

$$d_\gamma(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\left\{\frac{1}{6}\sum_{j=1}^{m}\sum_{i=1}^{n} w_i\left[\left|\Delta^{\mathcal{L}}_{ij}\mathbb{T}\right|^\gamma+\left|\Delta^{\mathcal{U}}_{ij}\mathbb{T}\right|^\gamma+\left|\Delta^{\mathcal{L}}_{ij}\mathbb{F}\right|^\gamma+\left|\Delta^{\mathcal{U}}_{ij}\mathbb{F}\right|^\gamma\right]\right\}^{\frac{1}{\gamma}},$$

Where $\Delta^{\mathcal{L}}_{ij}\mathbb{T}=\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i)-\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)$, $\Delta^{\mathcal{U}}_{ij}\mathbb{T}=\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i)-\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)$, $\Delta^{\mathcal{L}}_{ij}\mathbb{F}=\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i)-\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)$ and $\Delta^{\mathcal{U}}_{ij}\mathbb{F}=\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i)-\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)$.

Normalized generalized interval vague distance is

$$d^n_\gamma(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\left\{\frac{1}{6n}\sum_{j=1}^{m}\sum_{i=1}^{n} w_i\left[\left|\Delta^{\mathcal{L}}_{ij}\mathbb{T}\right|^\gamma+\left|\Delta^{\mathcal{U}}_{ij}\mathbb{T}\right|^\gamma+\left|\Delta^{\mathcal{L}}_{ij}\mathbb{F}\right|^\gamma+\left|\Delta^{\mathcal{U}}_{ij}\mathbb{F}\right|^\gamma\right]\right\}^{\frac{1}{\gamma}},$$

If $w=\left\{\frac{1}{n},\frac{1}{n},\dots,\frac{1}{n}\right\}$, then

$$d_\gamma(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\left\{\frac{1}{6}\sum_{j=1}^{m}\sum_{i=1}^{n}\left[\left|\Delta^{\mathcal{L}}_{ij}\mathbb{T}\right|^\gamma+\left|\Delta^{\mathcal{U}}_{ij}\mathbb{T}\right|^\gamma+\left|\Delta^{\mathcal{L}}_{ij}\mathbb{F}\right|^\gamma+\left|\Delta^{\mathcal{U}}_{ij}\mathbb{F}\right|^\gamma\right]\right\}^{\frac{1}{\gamma}},$$

$$d_\gamma(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\left\{\frac{1}{6n}\sum_{j=1}^{m}\sum_{i=1}^{n}\left[\left|\Delta^{\mathcal{L}}_{ij}\mathbb{T}\right|^\gamma+\left|\Delta^{\mathcal{U}}_{ij}\mathbb{T}\right|^\gamma+\left|\Delta^{\mathcal{L}}_{ij}\mathbb{F}\right|^\gamma+\left|\Delta^{\mathcal{U}}_{ij}\mathbb{F}\right|^\gamma\right]\right\}^{\frac{1}{\gamma}}.$$

Particular case

I. If $\gamma=1$ then then following hamming distance and normalized hamming distance between (IVVSS)

$$d^H_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{\left[\left|\Delta^{\mathcal{L}}_{ij}\mathbb{T}\right|+\left|\Delta^{\mathcal{U}}_{ij}\mathbb{T}\right|+\left|\Delta^{\mathcal{L}}_{ij}\mathbb{F}\right|+\left|\Delta^{\mathcal{U}}_{ij}\mathbb{F}\right|\right]}{6},$$

$$d^{nH}_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\frac{d^H_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})}{mn},$$

II. If $\gamma=2$ then the equation reduced to the following Euclidean distance and normalized Euclidean distance between (IVVSS)

$$d^E_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{\left(\Delta^{\mathcal{L}}_{ij}\mathbb{T}\right)^2+\left(\Delta^{\mathcal{U}}_{ij}\mathbb{T}\right)^2+\left(\Delta^{\mathcal{L}}_{ij}\mathbb{F}\right)^2+\left(\Delta^{\mathcal{U}}_{ij}\mathbb{F}\right)^2}{6}},$$

$$d^{NE}_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\frac{d^E_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})}{\sqrt{mn}}.$$

4 | Similarity Measures for Vague Soft Sets

This section proposes several similarity measures of (IVVSSs)

Similarity measurements are a generalization of distance measures, as is widely known. As a result, we can define similarity measures using the provided distance measurements. So we can define some similarity meaures between IVNSSs $\widetilde{\mathbb{I}_1}=(\widetilde{\mathcal{S}_1},\mathbb{E})$ and $\widetilde{\mathbb{I}_2}=(\widetilde{\mathcal{S}_2},\mathbb{E})$ as follows:

Definition 23. Now for each $\mathfrak{s}i\in\mathbb{E}$, $\mathcal{S}_1(\mathfrak{s}_i)$ and $\mathcal{S}_2(\mathfrak{s}_i)$ are (IVSs). To find similarity between $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$. We first find the similarity between $\widetilde{\mathcal{S}_1}(\mathfrak{s}_i)$ and $\widetilde{\mathcal{S}_2}(\mathfrak{s}_i)$.

Based on the distance measures defined above the similarity as follows:

$S^{H}_{IVVSS}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = \frac{1}{1+d^{H}_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})}$ and $S^{\mathbb{E}}_{IVVSS}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = \frac{1}{1+d^{\mathbb{E}}_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})}$,

$S^{nH}_{IVVSS}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) \frac{1}{1+d^{nH}_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})}$ and $S^{n\mathbb{E}}_{IVVSS}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = \frac{1}{1+d^{nE}_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})}$,

Example 4. Based on *Example 3*, then

$$S^{H}_{IVVSS}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = \frac{1}{1+0.71} = \frac{1}{1.71} = 0.58.$$

we define similarity measure between (IVVSSs) $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$ as follows:

$$S_{DM}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = 1 - \left\{\frac{1}{6n}\sum_{i=1}^{n} \left[\left|\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i) - \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\right|^{\gamma} + \left|\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i) - \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\right|^{\gamma} + \left|\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i) - \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\right|^{\gamma} + |\ \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i) \ \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\right|^{\gamma}\right]\right\}^{\frac{1}{\gamma}}.$$

Where γ>0 and $S_{DM}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2})$ is the degree of similarity of $\widetilde{\mathcal{S}_1}$ and $\widetilde{\mathcal{S}_2}$.
If we take the weight of each element $\varkappa_i \in \mathbb{U}$ into account, then

$$S^{w}_{DM}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = 1 - \left\{\frac{1}{6}\sum_{i=1}^{n} w_i \left[\left[\left|\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})} - \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\right|^{\gamma} + \left|\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i) - \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\right|^{\gamma} + + \left|\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i) - \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\right|^{\gamma} + |\ \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i) - \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\right|^{\gamma}\right]\right]\right\}^{\frac{1}{\gamma}}.$$

$S_{DM}(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2})$ satisfies all the properties of definition.

$[|\Delta^{\mathcal{L}}_{ij}\mathbb{T}| + |\Delta^{\mathcal{U}}_{ij}\mathbb{T}| + |\Delta^{\mathcal{L}}_{ij}\mathbb{F}| + |\Delta^{\mathcal{U}}_{ij}\mathbb{F}|]$

Also we define another similarity measure of $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$ as:

$$S(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = 1 - \left[\frac{\sum_{i=1}^{n}\left(\left|\Delta^{\mathcal{L}}_{ij}\mathbb{T}\right|^{\gamma} + \left|\Delta^{\mathcal{U}}_{ij}\mathbb{T}\right|^{\gamma} + \left|\Delta^{\mathcal{L}}_{ij}\mathbb{F}\right|^{\gamma} + \left|\Delta^{\mathcal{U}}_{ij}\mathbb{F}\right|^{\lambda}\right)}{\sum_{i=1}^{n}\left(||\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i) + \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)\right|^{\gamma} + \left|\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i) + \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)\right|^{\gamma} + \left|\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i) + \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)\right|^{\gamma} + \left|\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i) + \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)\right|^{\gamma}\right)}\right]^{\frac{1}{\gamma}}.$$

If we take the weight of each element $\varkappa_i \in \mathbb{U}$ into account, then

$$S(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = 1 - \left[\frac{\sum_{i=1}^{n} w_i\left(\left|\Delta^{\mathcal{L}}_{ij}\mathbb{T}\right|^{\gamma} + \left|\Delta^{\mathcal{U}}_{ij}\mathbb{T}\right|^{\gamma} + \left|\Delta^{\mathcal{L}}_{ij}\mathbb{F}\right|^{\gamma} + \left|\Delta^{\mathcal{U}}_{ij}\mathbb{F}\right|^{\lambda}\right)}{\sum_{i=1}^{n} w_i\left(\left|\ \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i) + \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}}(\varkappa_1)\right|^{\gamma} + \left|\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i) + \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)\right|^{\gamma} + \left|\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}}(\varkappa_1) + \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)\right|^{\gamma} + \left|\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i) + \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)\right|^{\gamma}\right)}\right]^{\frac{1}{\gamma}}$$

Definition 24. Let $S_i(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2})$ indicates the similarity between the (IVVSSs) $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$. Similarity between $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$ first we have to find the similarity between their e approximations. Let $S_i(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2})$ denote the similarity between the two $\mathfrak{s}_i$ - approximations $\widetilde{\mathcal{S}_1}(\mathfrak{s}_i)$ and $\widetilde{\mathcal{S}_2}(\mathfrak{s}_i)$.

Let $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$ be two (IVVSSs) then we define a similarity measure between $\widetilde{\mathcal{S}_1}(\mathfrak{s}_i)$ and $\widetilde{\mathcal{S}_2}(\mathfrak{s}_i)$ as follows:

$$S_i(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = \frac{\sum_{i=1}^{n} \begin{cases} min\left\{\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s_i)}(\varkappa_j), \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s_i)}(\varkappa_j)\right\} + min\left\{\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s_i)}(\varkappa_j), \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s_i)}(\varkappa_j)\right\} + \\ min\left\{\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s_i)}(\varkappa_j), \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s_i)}(\varkappa_j)\right\} + min\left\{\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s_i)}(\varkappa_j), \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s_i)}(\varkappa_j)\right\} \end{cases}}{\sum_{i=1}^{n} \begin{cases} max\left\{\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s_i)}(\varkappa_j), \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s_i)}(\varkappa_j)\right\} + max\left\{\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s_i)}(\varkappa_j), \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s_i)}(\varkappa_j)\right\} + \\ max\left\{\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s_i)}(\varkappa_j), \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s_i)}(\varkappa_j)\right\} + max\left\{\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s_i)}(\varkappa_j), \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s_i)}(\varkappa_j)\right\} \end{cases}},$$

Then $S(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = max_i S_i(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2})$.

Proposition 1. Let $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$ be (IVVSSs) then

I. $0 \leq S(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) \leq 1$.

II. $S(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = S(\widetilde{\mathbb{I}_2}, \widetilde{\mathbb{I}_1})$.

III. $S(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) = 1$ if $\widetilde{\mathbb{I}_1} = \widetilde{\mathbb{I}_2}$.

$\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i) = \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i), \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i) = \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i) and\ \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i) = \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i) = \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)$ for $i = 1,2, \dots, n$

Iv. $\widetilde{\mathbb{I}_1} \subseteq \widetilde{\mathbb{I}_2} \subseteq \widetilde{\mathbb{I}_3} \Rightarrow S(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}) \leq min(S(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}), S(\widetilde{\mathbb{I}_2}, \widetilde{\mathbb{I}_3})$.

Proof. Properties (I) and (II) follows from definition

III. it is clearly that if $\widetilde{\mathbb{I}_1} = \widetilde{\mathbb{I}_2} \Rightarrow S\left(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}^{\top}\right) = 1$,

$$\Rightarrow \sum_{i=1}^{I} \left\{min\left[\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_{1}}_{(s)}}(\varkappa_i), \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} + min\left\{\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} +\right.$$

$$+min\left\{\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} + min\left\{\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\}$$

$$= \sum_{i=1}^{n} \left\{max\left\{\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} + max\left\{\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} +\right.$$

$$+max\left\{\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} + max\left\{\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\}$$

$$\Rightarrow \sum_{i=1}^{n} \left\{\left[min\left\{\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} - max\left\{\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\}\right] +\right.$$

$$\left[min\left\{\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} - max\left\{\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\}\right]$$

$$+ \left[min\left\{\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} - max\left\{\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\}\right] +$$

$$\left.\left[min\left\{\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} - max\left\{\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\}\right]\right\} = 0.$$

Thus for each,

$$\left[min\left\{\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} - max\left\{\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\}\right] = 0$$

$$\left[min\left\{\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(x_E), \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} - max\left\{\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\}\right] = 0$$

$$\left[min\left\{\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} - max\left\{\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\}\right] = 0$$

$$\left[min\left\{\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\} - max\left\{\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(x_i), \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i)\right\}\right] = 0 \text{ holds}$$

Thus $\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i) = \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i), \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i) = \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i), \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_3}}(\varkappa_i) = \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_4}}(\varkappa_i)$ and $\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i) = \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i) \Rightarrow \widetilde{\mathbb{I}_1} = \widetilde{\mathbb{I}_2}$

IV. Now we prove the last result and for this Let $\widetilde{\mathbb{I}_1} \subseteq \widetilde{\mathbb{I}_2} \subseteq \widetilde{\mathbb{I}_3}$, then we have

$\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i) \leq \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i) \leq \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_4}}(\varkappa), \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa_i) \leq \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa_i) \leq \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_5}}(\varkappa), \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa) \geq \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa) \geq \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_4}}(\varkappa), \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(s)}(\varkappa) \geq \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(s)}(\varkappa) \geq \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_4}}(\varkappa)$ for all $\varkappa \in \mathbb{U}$,

now

$\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}}(\varkappa) + \mathcal{U}(\varkappa) + \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}}(\varkappa) + \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}}(\varkappa) \geq \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}}(\varkappa) + \mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}}(\varkappa) + \mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_4}}(\varkappa) + \mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_4}}(\varkappa)$, and

$$\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_2}(\varkappa) + \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_2}(\varkappa) + \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_1}(\varkappa) + \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_1}(\varkappa) \geq \mathbb{T}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa) + \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa) + \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_1}(\varkappa) + \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_1}(\varkappa),$$

$$S(\widetilde{\mathbb{I}}_1, \widetilde{\mathbb{I}}_2) = \frac{\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_1}(\varkappa)+\mathbb{T}^{\mathcal{U}}_{\widetilde{S}_1}(\varkappa)+\mathbb{F}^{\mathcal{L}}_{\widetilde{S}_2}(\varkappa)+\mathbb{F}^{\mathcal{U}}_{\widetilde{S}_2}(\varkappa)}{\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_2}(\varkappa)+\mathbb{T}^{\mathcal{U}}_{\widetilde{S}_2}(\varkappa)+\mathbb{F}^{\mathcal{L}}_{\widetilde{S}_1}(\varkappa)+\mathbb{F}^{\mathcal{U}}_{\widetilde{S}_1}(\varkappa)} \geq \frac{\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_1}(\varkappa)+\mathbb{T}^{\mathcal{U}}_{\widetilde{S}_1}(\varkappa)+\mathbb{F}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa)+\mathbb{F}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa)}{\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa)+\mathbb{T}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa)+\mathbb{F}^{\mathcal{L}}_{\widetilde{S}_1}(\varkappa)+\mathbb{F}^{\mathcal{U}}_{\widetilde{S}_1}(\varkappa)} = S(\widetilde{\mathbb{I}}_1, \widetilde{\mathbb{I}}_3).$$

Again similarly we have

$$\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_2}(\varkappa) + \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_2}(\varkappa) + \mathrm{F}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa) + \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa) \geq \mathbb{T}(\varkappa) + \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_1}(\varkappa) + \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa) + \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa),$$

$$\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa) + \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa) + \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_1}(\varkappa) + \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_1}(\varkappa) \geq \mathbb{T}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa) + \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa) + \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_2}(\varkappa) + \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_2}(\varkappa),$$

$$S(,\Omega) = \frac{\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_2}(\varkappa) + \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_5}(\varkappa) + \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa) + \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa)}{\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa) + \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa) + \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_2}(\varkappa) + \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_2}(x)} \geq \frac{\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_5}(\varkappa) + \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_1}(\varkappa) + \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa) + \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa)}{\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa) + \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa) + \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_1}(\varkappa) + \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_1}(\varkappa)}$$
$$= S(\widetilde{\mathbb{I}}_1, \widetilde{\mathbb{I}}_3),$$

$$\Rightarrow S(\widetilde{\mathbb{I}}_1, \widetilde{\mathbb{I}}_3) \leq min\left(S(\widetilde{\mathbb{I}}_1, \widetilde{\mathbb{I}}_2), S(\widetilde{\mathbb{I}}_2, \widetilde{\mathbb{I}}_2)\right).$$

If we take the weight of each element $\varkappa_i \in \mathbb{U}$ into account, then $S(\widetilde{\mathbb{I}}_1, \widetilde{\mathbb{I}}_2) =$

$$\frac{\sum_{i=1}^{n} w_i \left\{ \begin{array}{l} min\left\{\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_3}(\varkappa_i), \mathbb{T}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa_i)\right\} + min\left\{\mathbb{T}^{\mathcal{U}}_{\widetilde{S}_3}(\varkappa_i), \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa_i)\right\} \\ + min\left\{\mathbb{F}^{\mathcal{L}}_{\widetilde{S}_3}(\varkappa_i), F^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa_i)\right\} + min\left\{\mathbb{F}^{\mathcal{U}}_{\widetilde{S}_3}(\varkappa_i), \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa_i)\right\} \end{array} \right.}{\sum_{i=1}^{n} w_i \left\{ \begin{array}{l} max\left\{\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_3}(\varkappa_i), \mathbb{T}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa_i)\right\} + max\left\{\mathbb{T}^{\mathcal{U}}_{\widetilde{S}_3}(\varkappa_i), \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa_i)\right\} + \\ max\left\{\mathbb{F}^{\mathcal{L}}_{\widetilde{S}_3}(\varkappa_i), \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa_i)\right\} + max\left\{\mathbb{F}^{\mathcal{U}}_{\widetilde{S}_3}(\varkappa_i), \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa_i)\right\} \end{array} \right.}.$$

Theorem 3. Similarity measure based for matching function by using (IVVVSs):

Let $\widetilde{\mathbb{I}}_1$ and $\widetilde{\mathbb{I}}_2$be two (IVVSSs), then we define a similarity measure between $\widetilde{\mathbb{I}}_1$ and $\widetilde{\mathbb{I}}_2$ as follows:

$$S_i(\widetilde{\mathbb{I}}_1, \widetilde{\mathbb{I}}_2) = \frac{\sum_{i=1}^{n} \left(\begin{array}{l} \left(\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_3}(\varkappa_i) \cdot \mathbb{T}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa_i)\right) + \left(\mathbb{T}^{\mathcal{U}}_{\widetilde{S}_3}(\varkappa_i) \cdot \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa_i)\right) \\ + \left(\mathbb{F}^{\mathcal{L}}_{\widetilde{S}_3}(\varkappa_i) \cdot \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa_i)\right) + \left(\mathbb{F}^{\mathcal{U}}_{\widetilde{S}_3}(\varkappa_i) \cdot \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa_i)\right) \end{array} \right)}{max \left(\begin{array}{l} \sum_{i=}^{n} = \left(\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_3}(\varkappa_i)^2 + \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_3}(\varkappa_i)^2 + \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_3}(\varkappa_i)^2 + \mathbb{F}^{\mathcal{U}}_{3}(\varkappa_i)^2 \right), \\ \sum_{i}^{n} = \left(\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa_i)^2 + \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa_i)^2 + \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_4}(\varkappa_i)^2 + \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_4}(\varkappa_i)^2 \right) \end{array} \right)},$$

$$\mathbb{T}^{\mathcal{L}}_{\widetilde{S}_{1(s)}}(\varkappa_i) = \mathbb{T}^{\mathcal{L}}_{\widetilde{S}_{2(s)}}(\varkappa_i), \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_{1(s)}}(\varkappa_i) = \mathbb{T}^{\mathcal{U}}_{\widetilde{S}_{2(s)}}(\varkappa_i),$$

and

$$\mathbb{F}^{\mathcal{L}}_{\widetilde{S}_{1(s)}}(\varkappa_i) = \mathbb{F}^{\mathcal{L}}_{\widetilde{S}_{2(s)}}(\varkappa_i), \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_{1(s)}}(\varkappa_i) = \mathbb{F}^{\mathcal{U}}_{\widetilde{S}_{2(s)}}(\varkappa_i).$$

Proof. i. $0 \leq S_{\widetilde{S}_2 F}(\widetilde{\mathbb{I}}_1, \widetilde{\mathbb{I}}_2) \leq 1$.

The inequality $S_{\widetilde{S}_2 F}(\widetilde{\mathbb{I}}_1, \widetilde{\mathbb{I}}_2) \geq 0$ is obvious.Thus,we only prove the inequality $S(\widetilde{\mathbb{I}}_1, \widetilde{\mathbb{I}}_2) \leq 1$.

$$S_{\widetilde{\mathcal{S}_2}\mathrm{F}}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2}) = \sum_{i=1}^{n}\left(\left(\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\cdot\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\right)+\left(\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i)\cdot\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\right)\right.$$

$$\left.+\left(\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i)\cdot\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\right)+\left(\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i)\cdot\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)\right)\right)$$

$$= \mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_1)\cdot\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_1)+\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_2)\cdot\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_2)+\cdots+\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_n)\cdot\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_n)+\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_1)\cdot$$
$$\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_1)+\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_2)\cdot\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_2)+\cdots+\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_n)\cdot\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_n)+$$

$$\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_1)\cdot\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_1)+\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_2)\cdot\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_2)+\cdots+\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_n)\cdot\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_n)+\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_1)\cdot$$
$$\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_1)+\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_2)\cdot\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_2)+\cdots+\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_n)\cdot\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_n).$$

According to the Cauchy-Schwarz inequality:

$$(\varkappa_1\cdot y_1+\varkappa_2\cdot y_2+\cdots+\varkappa_n\cdot y_n)^2\le(\varkappa_1^2+\varkappa_2^2+\cdots+\varkappa_n^2)\cdot(y_1^2+y_2^2+\cdots+y_n^2)$$

where $(\varkappa_1,\varkappa_2,\dots,\varkappa_n)\in R^n$ and $(y_1,y_2,\dots,y_n)\in R^n$ we can obtain

$$\left[S_{\widetilde{\mathcal{S}_2}F}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})\right]^2\le\sum_{i=1}^{n}\left(\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i)^2+\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i)^2+\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i)^2\right.$$
$$\left.+\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}(\mathfrak{s})}(\varkappa_i)^2\right).$$
$$\sum_{i=1}^{n}\left(\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)^2+\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)^2+\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)^2+\right.$$
$$\left.\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}(\mathfrak{s})}(\varkappa_i)^2\right)=S(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_1})\cdot S(\widetilde{\mathbb{I}_2},\widetilde{\mathbb{I}_2}).$$

Thus $S_{\widetilde{\mathcal{S}_2}\mathbb{F}}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})\le[S(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_1})]^{\frac{1}{2}}\cdot[S(\widetilde{\mathbb{I}_2},\widetilde{\mathbb{I}_2})]^{\frac{1}{2}}$,

Then $S_{\widetilde{\mathcal{S}_2}\mathrm{F}}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})\le max\{S(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_1}),S(\widetilde{\mathbb{I}_2},\widetilde{\mathbb{I}_2})\ ]$.

Therefore, $S_{\widetilde{\mathcal{S}_2}\mathbb{F}}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})\le 1$.

If we take the weight of each element $\varkappa_i\in\mathbb{U}$ into account, then $S^{W}_{\widetilde{\mathcal{S}_2}\mathrm{F}}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=$

$$\frac{\sum_{i=1}^{n}w_i\begin{pmatrix}\left(\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i)\cdot\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)\right)+\left(\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i)\cdot\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)\right)+\\\left(\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i)\cdot\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)\right)+\left(\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i)\cdot\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)\right)\end{pmatrix}}{max\begin{pmatrix}\sum_{i}^{n}=w_i\left(\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i)^2+\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i)^2+\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i)^2+\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_1}}(\varkappa_i)^2\right),\\\sum_{i}^{n}=w_i\left(\mathbb{T}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)^2+\mathbb{T}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)^2+\mathbb{F}^{\mathcal{L}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)^2+\mathbb{F}^{\mathcal{U}}_{\widetilde{\mathcal{S}_2}}(\varkappa_i)^2\right)\end{pmatrix}}.$$

Application 2. Let $\mathbb{U}=\{yes=\varkappa_1, no=\varkappa_2\}$. Here set of parameters $\mathbb{E}$ is the set of certain visible symptoms. Let $\mathbb{E}=\{\mathfrak{s}_1,\mathfrak{s}_2,\mathfrak{s}_3,\mathfrak{s}_4,\mathfrak{s}_5,\mathfrak{s}_6\}$, where $\mathfrak{s}_1$= high body temperature, $\mathfrak{s}_2$ = cough with chest mobbing, $\mathfrak{s}_3$ = body ache, $\mathfrak{s}_4$ = headache, $\mathfrak{s}_5$ = loose motion, and $\mathfrak{s}_6$ = breathing trouble. Our model inv-soft for pneumonia γ is given below and this can be prepared with the help of a medical person:

$$\widetilde{\mathbb{I}_1}=\begin{bmatrix}\left(\left(\mathfrak{s}_1,\begin{pmatrix}<\varkappa_1,[05\times10^{-1},06\times10^{-1}],[03\times10^{-1},04\times10^{-1}]>,\\<\varkappa_2,[05\times10^{-1},06\times10^{-1}],[03\times10^{-1},04\times10^{-1}]>,\end{pmatrix}\right)\right),\\\left(\left(\mathfrak{s}_2,\begin{pmatrix}<\varkappa_1,[05\times10^{-1},06\times10^{-1}],[03\times10^{-1},04\times10^{-1}]>,\\<\varkappa_2,[02\times10^{-1},03\times10^{-1}],[03\times10^{-1},06\times10^{-1}]>,\end{pmatrix}\right)\right),\\\left(\left(\mathfrak{s}_3,\begin{pmatrix}<\varkappa_1,[04\times10^{-1},06\times10^{-1}],[02\times10^{-1},03\times10^{-1}]>,\\<\varkappa_2,[05\times10^{-1},06\times10^{-1}],[03\times10^{-1},04\times10^{-1}]>,\end{pmatrix}\right)\right),\\\left(\left(\mathfrak{s}_4,\begin{pmatrix}<\varkappa_1,[03\times10^{-1},04\times10^{-1}],[02\times10^{-1},04\times10^{-1}]>,\\<\varkappa_2,[02\times10^{-1},05\times10^{-1}],[04\times10^{-1},05\times10^{-1}]>,\end{pmatrix}\right)\right),\\\left(\left(\mathfrak{s}_5,\begin{pmatrix}<\varkappa_1,[05\times10^{-1},06\times10^{-1}],[03\times10^{-1},04\times10^{-1}]>,\\<\varkappa_2,[04\times10^{-1},06\times10^{-1}],[03\times10^{-1},04\times10^{-1}]>,\end{pmatrix}\right)\right),\\\left(\left(\mathfrak{s}_6,\begin{pmatrix}<\varkappa_1,[04\times10^{-1},06\times10^{-1}],[02\times10^{-1},03\times10^{-1}]>,\\<\varkappa_2,[03\times10^{-1},04\times10^{-1}],[01\times10^{-1},04\times10^{-1}]>,\end{pmatrix}\right)\right)\end{bmatrix}.$$

The sick person suffers from a fever, cough, and headache. Following our conversation with him, we can create his (IVVSSs) $\widetilde{\mathbb{I}_2}$ as follows:

$$\widetilde{\mathbb{I}_2}=\begin{bmatrix}\left(\left(\varsigma_1,\begin{pmatrix}<\varkappa_1,[01\times10^{-1},02\times10^{-1}],[08\times10^{-1},09\times10^{-1}]>,\\<\varkappa_2,[01\times10^{-1},02\times10^{-1}],[08\times10^{-1},09\times10^{-1}]>,\end{pmatrix}\right)\right),\\\left(\left(\varsigma_2,\begin{pmatrix}<\varkappa_1,[08\times10^{-1},09\times10^{-1}],[08\times10^{-1},09\times10^{-1}]>,\\<\varkappa_2,[02\times10^{-1},03\times10^{-1}],[03\times10^{-1},06\times10^{-1}]>,\end{pmatrix}\right)\right),\\\left(\left(\varsigma_3,\begin{pmatrix}<\varkappa_1,[01\times10^{-1},09\times10^{-1}],[06\times10^{-1},09\times10^{-1}]>,\\<\varkappa_2,[01\times10^{-1},08\times10^{-1}],[08\times10^{-1},07\times10^{-1}]>,\end{pmatrix}\right)\right),\\\left(\left(\varsigma_4,\begin{pmatrix}<\varkappa_1,[08\times10^{-1},08\times10^{-1}],[03\times10^{-1},03\times10^{-1}]>,\\<\varkappa_2,[06\times10^{-1},09\times10^{-1}],[08\times10^{-1},09\times10^{-1}]>,\end{pmatrix}\right)\right),\\\left(\left(\varsigma_5,\begin{pmatrix}<\varkappa_1,[03\times10^{-1},04\times10^{-1}],[08\times10^{-1},08\times10^{-1}]>,\\<\varkappa_2,[05\times10^{-1},09\times10^{-1}],[01\times10^{-1},02\times10^{-1}]>,\end{pmatrix}\right)\right),\\\left(\left(\varsigma_6,\begin{pmatrix}<\varkappa_1,[01\times10^{-1},02\times10^{-1}],[07\times10^{-1},07\times10^{-1}]>,\\<\varkappa_2,[07\times10^{-1},08\times10^{-1}],[00\times10^{-1},04\times10^{-1}]>,\end{pmatrix}\right)\right)\end{bmatrix},$$

$$d^{H}_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\begin{Bmatrix}\frac{(|5-1|+|6-2|+|3-8|+|4-9|)\times10^{-1}}{6}+\frac{(|5-1|+|6-2|+|3-8|+|4-9|)\times10^{-1}}{6}+\\\frac{(|5-8|+|6-9|+|3-8|+|4-9|)\times10^{-1}}{6}+\frac{(|2-2|+|3-3|+|3-3|+|6-6|)\times10^{-1}}{6}+\\\frac{(|4-1|+|6-9|+|2-6|+|3-9|)\times10^{-1}}{6}+\frac{(|5-1|+|6-8|+|3-8|+|4-7|)\times10^{-1}}{6}+\\\frac{(|3-8|+|4-8|+|2-3|+|4-3|)\times10^{-1}}{6}+\frac{(|2-6|+|5-9|+|4-8|+|5-9|)\times10^{-1}}{6}+\\\frac{(|5-3|+|6-4|+|3-8|+|4-8|)\times10^{-1}}{6}+\frac{(|4-5|+|6-9|+|3-1|+|4-2|)\times10^{-1}}{6}+\\\frac{(|4-1|+|6-2|+|2-7|+|3-7|)\times10^{-1}}{6}+\frac{(|3-7|+|4-8|+|1-0|+|4-4|)\times10^{-1}}{6}\end{Bmatrix},$$

$$d^{H}_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\begin{Bmatrix}\frac{(4+4+5+5)\times10^{-1}}{6}+\frac{(4+4+5+5)\times10^{-1}}{6}+\\\frac{(3+3+5+5)\times10^{-1}}{6}+\frac{(0+0+0+0)\times10^{-1}}{6}+\\\frac{(3+3+4+6)\times10^{-1}}{6}+\frac{(4+2+5+3)\times10^{-1}}{6}+\\\frac{(5+4+1+1)\times10^{-1}}{6}+\frac{(4+1+4+4)\times10^{-1}}{6}+\\\frac{(2+2+5+4)\times10^{-1}}{6}+\frac{(1+3+2+2)\times10^{-1}}{6}+\\\frac{(3+4+5+4)\times10^{-1}}{6}+\frac{(4+4+1+0)\times10^{-1}}{6}\end{Bmatrix},$$

Then, we calculate the similarity measure between these two (IVVSSs) as follows:

$$S^{H}_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})=\frac{1}{1+d^{H}_{IVVSS}(\widetilde{\mathbb{I}_1},\widetilde{\mathbb{I}_2})}=0.28.$$

Hence, the two (IVVSSs), i.e., two symptoms $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_2}$ are not significantly similar. Therefore, we conclude that the person is not possibly suffering from pneumonia. A person suffering from the following symptoms whose corresponding (IVVSSs) $\widetilde{\mathbb{I}_3}$ is given below:

As a result, the two (IVVSSs), i.e., the two symptoms $\widetilde{\mathbb{I}_1}$and $\widetilde{\mathbb{I}_2}$, are not statistically significant. As a result, we determine that the individual is not suffering from pneumonia. An individual who exhibits the following symptoms and has the appropriate (IVVSSs) $\widetilde{\mathbb{I}_2}$:

$$\widetilde{\mathbb{I}_3} = \begin{bmatrix} \left(\left(\mathfrak{s}_1, \begin{pmatrix} < \varkappa_1, [05\times 10^{-1}, 07\times 10^{-1}], [03\times 10^{-1}, 05\times 10^{-1}] >, \\ < \varkappa_2, [06\times 10^{-1}, 06\times 10^{-1}], [03\times 10^{-1}, 05\times 10^{-1}] >, \end{pmatrix}\right)\right), \\ \left(\left(\mathfrak{s}_2, \begin{pmatrix} < \varkappa_1, [05\times 10^{-1}, 07\times 10^{-1}], [03\times 10^{-1}, 04\times 10^{-1}] >, \\ < \varkappa_2, [02\times 10^{-1}, 04\times 10^{-1}], [02\times 10^{-1}, 07\times 10^{-1}] >, \end{pmatrix}\right)\right), \\ \left(\left(\mathfrak{s}_3, \begin{pmatrix} < \varkappa_1, [04\times 10^{-1}, 07\times 10^{-1}], [01\times 10^{-1}, 03\times 10^{-1}] >, \\ < \varkappa_2, [04\times 10^{-1}, 08\times 10^{-1}], [02\times 10^{-1}, 08\times 10^{-1}] >, \end{pmatrix}\right)\right), \\ \left(\left(\mathfrak{s}_4, \begin{pmatrix} < \varkappa_1, [03\times 10^{-1}, 04\times 10^{-1}], [02\times 10^{-1}, 06\times 10^{-1}] >, \\ < \varkappa_2, [02\times 10^{-1}, 05\times 10^{-1}], [04\times 10^{-1}, 05\times 10^{-1}] >, \end{pmatrix}\right)\right), \\ \left(\left(\mathfrak{s}_5, \begin{pmatrix} < \varkappa_1, [05\times 10^{-1}, 06\times 10^{-1}], [03\times 10^{-1}, 04\times 10^{-1}] >, \\ < \varkappa_2, [04\times 10^{-1}, 06\times 10^{-1}], [01\times 10^{-1}, 08\times 10^{-1}] >, \end{pmatrix}\right)\right), \\ \left(\left(\mathfrak{s}_6, \begin{pmatrix} < \varkappa_1, [04\times 10^{-1}, 07\times 10^{-1}], [02\times 10^{-1}, 08\times 10^{-1}] >, \\ < \varkappa_2, [05\times 10^{-1}, 02\times 10^{-1}], [02\times 10^{-1}, 05\times 10^{-1}] > \end{pmatrix}\right)\right) \end{bmatrix},$$

then,

$$\mathrm{S}^{\mathrm{H}}_{\mathrm{IVVSS}}\left(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_3}\right) = \frac{1}{1+\mathrm{d}^{\mathrm{H}}_{\mathrm{IVVSS}}\left(\widetilde{\mathbb{I}_1}, \widetilde{\mathbb{I}_2}\right)} = 0.412,$$

In this case, the two (IVVSSs), i.e., symptoms $\widetilde{\mathbb{I}_1}$ and $\widetilde{\mathbb{I}_3}$, are very identical. As a result, we deduce that the person may have contracted pneumonia. This is only a simple example of how this method could be used to diagnose diseases, but it could be improved by including clinical data and other competing diagnoses.

5 | Conclusion

Measures are different techniques used in the field of mathematics. An entropy measure and distance similarity measures in the atmosphere of vague soft sets are addressed. Entropy and distance similarity measures are introduced in vague soft sets concerning crisp points of the spaces. After doing this, some more fundamental results are addressed, which are proved in a vague soft atmosphere concerning crisp points of the spaces. A better impression of these results is shown by planting suitable examples. Entropy and distance similarity measures are studied over the crisp points of the spaces. Lastly, distance and similarity measures of interval-valued vague soft sets are addressed. We'll pretend to generate identical circumstances in vague soft sets concerning soft points of the spaces in the future.

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Conflicts of Interest

The authors declare no conflict of interest.

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