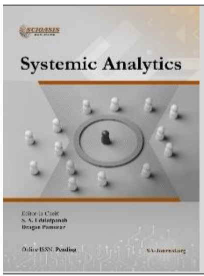

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A Paradigm Shift in Linear Programming: An Algorithm without Artificial Variables

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Abstract

Linear Programming (LP) is pivotal in operations research across various domains. The standard simplex method, while effective, faces challenges initializing when inequality constraints exist, often necessitating artificial variables. This paper presents a paradigm-shifting approach—eliminating artificial variables. The new method simplifies LP by leveraging negative and positive variables, saving significant time and resources compared to traditional Two-Phase and Big-M methods. A numerical example confirms our approach's superior efficiency and speed. This innovation promises to transform LP problem-solving, eliminating artificial variable burdens and streamlining computations.

Keywords: Linear programming, Artificial variables, Simplex method, Big-M method, Computational efficiency, Two-Phase process, Mathematical modeling.

1 | Introduction

Empirical assessments affirm that Linear Programming (LP) occupies a paramount position within the expansive domain of operations research. It's a striking revelation that many real-world predicaments can be seamlessly transmuted into LP models, thereby establishing this framework as an ineluctable cornerstone in contemporary applications spanning diverse domains, including energy management, military strategy, transportation logistics, and precision manufacturing optimization.

The seminal Dantzig's Simplex method, incubated within the crucible of the American Air Force during the tumultuous era of the Second World War [1], stands firmly at the forefront of productive solution techniques

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for navigating the labyrinthine intricacies of LP Problems (LPPs). The Simplex algorithm has solidified its standing as a preeminent stalwart in the pantheon of LP methodologies, meriting even the distinguished recognition of being included in the revered list of top ten algorithms of the twentieth century by the esteemed journal computing in science and engineering [2].

Notwithstanding its laurels, the path through the computational terrain of LP, guided by the Simplex algorithm, remains rife with inherent complexities. In the customary modus operandi of the Simplex algorithm, the odyssey invariably commences from a foundational vantage point known as the Basic Feasible Solution (BFS). However, this journey is underpinned by a stringent prerequisite: the positivity of the solution column or Right Hand Side (RHS) of the constraints. In stark contrast, when confronted with constraints adorned with the mantle of either a ≥ or an = sign, we find ourselves entangled in an intricate problem. Here, the initiation of the quest for the elusive BFS proves to be a daunting enigma, rendering the labyrinthine landscape of the simplex method even more perplexing. This presents a problem, as the labyrinthine maze of the simplex method conceals the path to an initial BFS when such constraints hold sway. In response to this enigma, a potent conceptual entity emerges from the shadows: the artificial variable. This conceptual sentinel takes center stage within the given LPP context, orchestrating a transformation that beckons the deployment of specialized techniques such as the Two-Phase or Big-M method [1].

While these techniques exhibit considerable intrigue and extensive applicability, they are burdened by substantial computational expenses. Researchers are directing heightened scrutiny toward minimizing computational time and costs as these methods find their way into real-world scenarios. Consequently, earnest endeavors have been undertaken to eschew the necessity of artificial variables within the realm of the simplex method.

Arsham [3] and Arsham et al. [4] introduced an algorithm devoid of artificial variables called Push-and-Pull. Junior and Lins [5] proposed a technique that enhances iteration efficiency by approximately 33% by furnishing the Simplex method with a more proficient initial basis. By employing the cosine criterion, Corley et al. [6] devised an algorithm that surpasses the standard simplex method in terms of efficiency. Nabli [7] introduced a novel initializing simplex method termed the Non-Feasible Basis (NFB) method. Stojković et al. [8] conducted a comparative analysis of three distinct methods for identifying a BFS through numerical test examples. Their findings showcased that two of these methods outperform the classical algorithm for deriving initial solutions, particularly on the Netlib test problems. Boonperm and Sinapiromsaran [9] unveiled a technique for non-acute constraint relaxation. This innovation not only obviates the need for artificial variables but also reduces the startup time required to resolve the initial relaxation problem. Their research demonstrated the superiority of the new algorithm over the original Simplex method, replete with artificial variables, mainly when applied to LPPs characterized by a substantial number of acute constraints. Saito et al. [10] expounded upon an active-set, cutting-plane approach named Constraint Optimal Selection Techniques (COSTs). They developed a new COST tailored to solve nonnegative LPPs.

Additionally, they introduced a geometric interpretation of this innovative selection rule and conducted computational comparisons, pitting the new COST against existing LP algorithms on sizeable sample problems. Gao [11] proposed enhancing Arsham's algorithm [3] and optimizing its performance. This enhancement entails a systematic search for non-basic variables within the Basic Variable Set (BVS), column by column, in a single pivot sequence from commencement to conclusion. This approach markedly reduces the computational time typically consumed by numerous repeated search sequences after each iteration. For further exploration, refer to [12–22]. Notably, the structural underpinnings of most of these methods are rooted in negative variables. When the count of artificial variables assumes significance, these methods offer an advantageous edge in terms of computational efficiency compared to the Two-phase and Big-M methods.

In LP, a paradigm shift occurs with the development of an algorithm that eliminates the need for artificial variables. This algorithm represents a significant advancement in the field as it simplifies the optimization process and enhances decision-making capabilities. Within the confines of this document, we introduce an innovative method designed to tackle LP challenges without the reliance on artificial variables. What distinguishes our approach is its foundational structure, which seamlessly incorporates both negative and positive variables, offering an elegantly straightforward implementation process. To highlight the effectiveness of our method, we present a numerical example, unraveling the results obtained through comprehensive discussion and analysis.

2 | Prerequisite

We discuss fundamental notation and initial findings that we will revisit later. For a more comprehensive examination, please consult [1].

The conventional formulation of LP, encompassing m constraints and n variables, can be articulated as follows:

$$\begin{aligned} \text{Min} \quad & z = \mathbf{cx}, \\ \text{s.t.} \quad & A\mathbf{x} = \mathbf{b}, \\ & \mathbf{x} \geq \mathbf{0}. \end{aligned} \tag{1}$$

Where $\text{rank}(A, b) = \text{rank}(A) = m$ and A is an $m \times n$ matrix. After carrying out permutations on the column of A, let $A = (B, N)$, where B is an $m \times m$ invertible matrix and is called basic matrix and N is an $m \times n - m$ matrix which is called non-basic matrix. Similarly, the solution $\mathbf{x}$ to the equation $A\mathbf{x} = \mathbf{b}$ can be arranged with respect to B, N, i.e. $\mathbf{x} = \begin{pmatrix} \mathbf{x}_B \\ \mathbf{x}_N \end{pmatrix}$, where

$$\begin{aligned} A\mathbf{x} = \mathbf{b} \Rightarrow (B, N)\begin{pmatrix} \mathbf{x}_B \\ \mathbf{x}_N \end{pmatrix} = \mathbf{b} \\ \Rightarrow B\mathbf{x}_B + N\mathbf{x}_N = \mathbf{b} \\ \Rightarrow \mathbf{x}_B = B^{-1}\mathbf{b} - B^{-1}N\mathbf{x}_N. \end{aligned} \tag{2}$$

Then

$$\begin{cases} \mathbf{x}_B = B^{-1}\mathbf{b} \geq \mathbf{0}, \\ \mathbf{x}_N = \mathbf{0}, \end{cases} \tag{3}$$

is called a Basic Solution (BS) of LP. This BS is said to be the BFS of the given problem when the condition $\mathbf{x}_B \geq \mathbf{0}$ met. The variables associated with BS are said to be basic variables, while others $\mathbf{x}_N$ are said to be non-basic variables. We express the basic variable from *Eqs. (1)* and *(2)* as:

$$\begin{aligned} \mathbf{x}_B &= B^{-1}\mathbf{b} - B^{-1}N\mathbf{x}_N \\ \Rightarrow \mathbf{x}_B &= B^{-1}\mathbf{b} - \sum_{j \in R} B^{-1}\mathbf{a}_j x_j \\ \Rightarrow \mathbf{x}_B &= \bar{\mathbf{b}} - \sum_{j \in R} (y_j) x_j. \end{aligned}$$

Here, R indicates the "current set of indices" of the non-basic variables and $\bar{\mathbf{b}} = B^{-1}\mathbf{b}$, $\mathbf{y}_j = B^{-1}\mathbf{a}_j$. Furthermore, the objective value of *Eq. (1)* is given by

$$
\begin{aligned}
z &= \mathbf{cx} \\
&= (\mathbf{c}_B, \mathbf{c}_N)\begin{pmatrix}\mathbf{x}_B \\ \mathbf{x}_N\end{pmatrix} \\
&= \mathbf{c}_B\mathbf{x}_B + \mathbf{c}_N\mathbf{x}_N \\
\Rightarrow z &= \mathbf{c}_B(\bar{\mathbf{b}} - \sum_{j\in R}\mathbf{y}_j x_j) + \sum_{j\in R} c_j x_j \\
\Rightarrow z &= \mathbf{c}_B\bar{\mathbf{b}} - \sum_{j\in R}\mathbf{c}_B\mathbf{y}_j x_j + \sum_{j\in R} c_j x_j \\
\Rightarrow z &= z_\circ - \sum_{j\in R}(z_j - c_j)x_j,
\end{aligned}
\tag{4}
$$

where $z_\circ = \mathbf{c}_B\bar{\mathbf{b}}$, $z_j = \mathbf{c}_B\mathbf{y}_j$. By the above demonstrations, we get the following LP:

$$
\begin{aligned}
\text{Min} \quad & z = z_\circ - \sum_{j\in R}(z_j - c_j)x_j, \\
\text{s.t.} \quad & \sum_{j\in R}\mathbf{y}_j x_j + \mathbf{x}_B = \bar{\mathbf{b}}, \\
& x_j \geq \circ, \quad j \in R, \\
& \mathbf{x}_B \geq \mathbf{0}.
\end{aligned}
\tag{5}
$$

3 | Simplex Algorithm Extension

Here, we present our Simplex Algorithm Extension (SAE) method. SAE is a groundbreaking approach designed to tackle LPPs without the need for artificial variables. Based on negative and positive variables, this innovative technique simplifies LP problem-solving while maintaining computational efficiency. Consider the problem:

$$
\begin{aligned}
\text{Min} \quad & z = \mathbf{cx}, \\
\text{s.t.} \quad & A\mathbf{x} \geq \mathbf{b}, \\
& \mathbf{x} \geq \mathbf{0}.
\end{aligned}
\tag{6}
$$

In which constraints can be written in the equality form by using slack variables as:

$$
\begin{aligned}
\text{Min} \quad & z = \mathbf{cx}, \\
\text{s.t.} \quad & -A\mathbf{x} + x_s = -\mathbf{b}, \\
& \mathbf{x}, x_s \geq \mathbf{0}.
\end{aligned}
\tag{7}
$$

From this model, it is seen that we cannot pick the starting basic for a BFS.

To do so, we present a new algorithm as follows.

Algorithm 1 (SAE).

While $\exists i$ s.t. $\bar{b}_i < 0$ do;

I. Choose a BS (for example $x_B = -\bar{\mathbf{b}}$).

II. Set $\bar{\mathbf{b}}_r = \text{Min}(\bar{\mathbf{b}}_i)$.

III. If for all j; $y_{rj} \geq 0$, stop; the considered LP is infeasible.

Otherwise, select x_k as entering variable by the following test:

$$\frac{\bar{b}_r}{y_{rk}} = \text{Max}\left\{\frac{\bar{b}_r}{y_{rj}} : y_{rj} < \circ\right\}.$$

IV. The index t of the blocking variable x_{B_t} which leaves the basis is determined by following:

$$\gamma_1 = \text{Min}\left\{\frac{\bar{b}_i}{y_{ik}} : \text{sign}(\bar{b}_i) = \text{sign}(y_{ik}) = \text{Positive}\right\},$$

$$\gamma_2 = \text{Max}\left\{\frac{\bar{b}_i}{y_{ik}} : \text{sign}(\bar{b}_i) = \text{sign}(y_{ik}) = \text{Negative}\right\},$$

$$\lambda = \frac{\bar{b}_t}{y_{tk}} = \text{Min}\{\gamma_1, \gamma_2\}.$$

V. Update the basis B where $\mathbf{a}_k$ (kth column of A) replaces $\mathbf{a}_t$ (tth column of A), update the index set R .

End while

The current basis is feasible.

Use the simplex algorithm.

4 | Analysis of SAE

In this section, we discuss all cases of SAE.

Case A: For $\bar{b}_i < 0$, if for all j; $y_{ij} \geq 0$, then LP has no feasible solution, because if there is an $\mathbf{x} \geq \mathbf{0}$, then we have

$$\sum_{j=1}^{n+m} a_{ij} x_j = \bar{b}_i.$$

Since for all j; $a_{ij} = y_{ij} \geq 0$ and $\mathbf{x}_j \geq \mathbf{0}$, therefore, the left-hand side of the above equation is $\geq \mathbf{0}$. But the right-hand side is negative. This contradiction shows that there could be no feasible solution to the LP.

We discuss all cases of our algorithm.

Case B: $\bar{b}_i < 0.$

Under this case, we have the following subcases:

Subcase B1: If $\exists k$ subject that: $y_{ik} \geq 0,$

Since

$$x_{B_i} = \underbrace{\bar{b}_i}_{\leq 0} - \lambda \underbrace{y_{ik}}_{\geq 0} \leq 0.$$

The component of x_{B_i} remains negative, and we need more repetition.

Subcase B2: If $\exists k$ subject that $y_{ik} < 0$ and the blocking variable be $\lambda = \gamma_1$.

In this case, the new value of x_{B_i} will be improved. Because

$$x_{B_i}^{New} = \underbrace{\bar{b}_i}_{\leq 0} - \lambda \underbrace{y_{ik}}_{< 0} \leq \bar{b}_i = x_{B_i}^{Old}.$$

Subcase B3: $y_{ik} < 0$ and the blocking variable is $\lambda = \gamma_2$.

In this case, the new value of x_{B_i} is nonnegative. Since $\lambda = \gamma_2$ we have

$$\lambda = \frac{\bar{b}_t}{y_{tk}} \geq \frac{\bar{b}_i}{y_{ik}}.$$

And since $y_{ik} < 0$, we get

$$0 \leq \bar{b}_i - y_{ik}\frac{\bar{b}_t}{y_{tk}} \Rightarrow x_{B_i} \geq 0.$$

Case C: $\bar{b}_i > 0.$

This case is further divided into four subcases.

Subcase C1: $y_{ik} \leq 0$

In this case, x_{B_i} is nonnegative.

$$x_{B_i} = \underbrace{\bar{b}_i}_{\geq 0} - \lambda \underbrace{y_{ik}}_{\leq 0} \geq 0.$$

Subcase C2: $y_{ik} > 0$ and the blocking variable is $\lambda = \gamma_1$.

Since $\lambda = \gamma_1$ we have

$$\lambda = \frac{\bar{b}_t}{y_{tk}} \leq \frac{\bar{b}_i}{y_{ik}}.$$

And since $y_{ik} > 0$, we get

$$0 \leq \bar{b}_i - y_{ik}\frac{\bar{b}_t}{y_{tk}} \Rightarrow x_{B_i} \geq 0.$$

Subcase C3: $y_{ik} > 0$ and the blocking variable is $\lambda = \gamma_1$.

In this case

$$\frac{\bar{b}_i}{y_{ik}} \geq \frac{\bar{b}_t}{y_{tk}} \Rightarrow \bar{b}_i - y_{ik}\frac{\bar{b}_t}{y_{tk}} \Rightarrow x_{B_i} \geq 0.$$

In general speaking, by the above demonstration, we can see that our method will lead the BS to the BFS.

5 | An Illustrative Example

Consider the following LP:

$$\begin{aligned}
\text{Min} \quad & z = 3x_1 + 2x_2 + 4x_3 - 6x_4, \\
\text{s.t.} \quad & \\
& x_1 + x_2 - x_3 - x_4 \geq 2, \\
& x_1 + 2x_2 - 3x_3 + 6x_4 \leq 1, \\
& 2x_1 - x_2 - 2x_3 + x_4 \geq 1, \\
& x_1, x_2, x_3, x_4 \geq 0.
\end{aligned}$$

By the SAE, we get

z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	-3	-2	-4	6	0	0	0	0
x_5	-1	-1	1	1	1	0	0	-2
x_6	1	2	-3	6	0	1	0	1
x_7	-2	1	2	-1	0	0	1	-1

$\bar{\mathbf{b}}_r = \bar{\mathbf{b}}_1 = -2$ and $x_k = x_1$ is the entering variable. Furthermore $\gamma_1 = 1, \gamma_2 = 2, \lambda = 1$. Therefore x_6 is the blocking variable, and we have

z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	0	4	-13	24	0	3	0	3
x_5	0	1	-2	7	1	1	0	-1
x_1	1	2	-3	6	0	1	0	1
x_7	0	5	-4	11	0	2	1	1

$\bar{\mathbf{b}}_r = \bar{\mathbf{b}}_1 = -1$ and $x_k = x_3$ is the entering variable. Furthermore $\gamma_1 = \{\}, \gamma_2 = 0.5, \lambda = 0.5$. Therefore x_5 is the blocking variable, and we have

z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	0	-2.5	0	-21.5	-7.5	-3.5	0	9.5
x_3	0	-0.5	1	-3.5	-0.5	-0.5	0	0.5
x_1	1	0.5	0	-2.5	-1.5	-0.5	0	2.5
x_7	0	3	0	-3	-2	0	1	3

Since $z_j - c_j \leq 0$ for all non-basic variables, and for all $i; \bar{\mathbf{b}}_i \geq 0$, the optimal point $x^* = (2.5, 0, 0.5, 0)$ with opjective $z^* = 9.5$ is reached. Here, we solve this example with Nabli's algorithm [7]. By using Nabli's Algorithm, we get

z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	-3	-2	-4	6	0	0	0	0
x_5	-1	-1	1	1	1	0	0	-2
x_6	1	2	-3	6	0	1	0	1
x_7	-2	1	2	-1	0	0	1	-1

$\bar{\mathbf{b}}_r = \bar{\mathbf{b}}_1 = -2$ and $x_k = x_2$ is the entering variable. Furthermore $\lambda = 2$. Therefore x_5 is the blocking variable and we have

z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	-1	0	-6	4	-3	0	0	4
x_2	1	1	-1	-1	-1	0	0	2
x_6	-1	0	-1	8	2	1	0	-3
x_7	-3	0	3	0	1	0	1	-3

$\bar{\mathbf{b}}_r = \bar{\mathbf{b}}_2 = -3$ and $x_k = x_1$ is the entering variable. Furthermore $\lambda = 3$. Therefore x_6 is the blocking variable and we have

z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	0	0	-5	-4	-5	-1	0	7
x_2	0	1	-2	7	1	1	0	-1
x_1	1	0	1	-8	-2	-1	0	3
x_7	0	0	6	-24	-5	-3	1	6

$\bar{\mathbf{b}}_r = \bar{\mathbf{b}}_1 = -1$ and $x_k = x_3$ is the entering variable. Furthermore $\lambda = 0.5$. Therefore x_2 is the blocking variable and we have

z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
	0	-2.5	0	-21.5	-7.5	-3.5	0	9.5
x_3	0	-0.5	1	-3.5	-0.5	-0.5	0	0.5
x_1	1	0.5	0	-2.5	-1.5	-0.5	0	2.5
x_7	0	3	0	-3	-2	0	1	3

Therefore, the optimal point $x^* = (2.5, 0, 0.5, 0)$ with the objective $z^* = 9.5$ is reached with three iterations. The classic methods, such as the Two-Phase and Big-M methods, have more computational requirements, see [7]. Therefore, the efficiency of SAE is better than the mentioned methods.

6 | Conclusions

This paper has set out with the primary objective of introducing a novel method for tackling the LPP. The pivotal innovation of this Algorithm lies in its deliberate omission of artificial variables, thereby significantly curtailing the computation time required to ascertain the optimal solution. Unlike the conventional approach, which invariably necessitates the introduction of artificial variables when faced with constraints bearing a greater than or equal sign, our method eradicates this prerequisite. We have introduced this novel algorithm and conducted a comprehensive analysis of its convergence properties. The practical applicability of our approach has been substantiated through a numerical example, revealing that our Algorithm outperforms classical methods, emerging as a more effective and expeditious solution to LP challenges.

Compliance with ethical standards

Conflict of Interest

The author declares no conflict of interest.

Ethical Approval

This article contains no studies with human participants or animals performed by the author.

Data Availability Statement

No data were used to support this study.

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