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Soft Intersection Almost Bi-ideals of Semigroups

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Abstract

Soft intersection bi-ideal is a generalization of soft intersection quasi-ideal; soft intersection quasi-ideal is a generalization of soft intersection left (right) ideal. In this study, to generalize nonnull soft intersection bi-ideals of semigroups, we introduce the concept of soft intersection almost bi-ideals and studied its basic properties in detail. By obtaining that if a nonempty set A is almost bi-ideal, then its soft characteristic function is soft intersection almost bi-ideal and vice versa, we acquire many interesting relationships between almost bi-ideals and soft intersection almost bi-ideals concerning minimality, primeness, semiprimeness, and strongly primeness.

Keywords: Soft set, Semigroup, Bi-ideal, Soft intersection bi-ideal, Soft intersection almost bi-ideal.

1 | Introduction

A semigroup is the fundamental algebraic structure in theoretical computer science, automata, coding theory, formal languages, graph theory, and optimization theory. Ideals are crucial to examining algebraic structures and their applications. The ideal is the basic concept for progressing mathematical structures and their applications. Dedekind proposed the idea of ideals for the study of algebraic numbers, and Noether generalized the concept of ideals to associative rings. Good and Hughes [1] established the notion of bi-ideals for semigroup in 1952. The idea of quasi-ideals was initially suggested by Steinfeld [2] for semigroups and subsequently for rings. The generalization of ideals is vital to encourage more investigation of mathematical structures. Numerous mathematicians presented distinctive developments of ideals illustrating imperative outcomes regarding characterizing algebraic structures. Whereas the bi-ideals are a generalization of quasi-ideals, the quasi-ideals are a generalization of left and right ideals.

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Also, the concept of almost left, right, and two-sided ideals of semigroups was first provided by Grošek and Satko [3] in 1980. As an extension of bi-ideals, Bogdanovic [4] proposed the concept of almost bi-ideals in semigroups later in 1981. In 2018, Wattanatripop et al. [5] proposed the perception of almost quasi-ideals by utilizing the notions of quasi-ideals of semigroups and almost ideals. Using the idea of almost ideals and interior ideals of semigroups, Kaopusek et al. [6] proposed the notions of almost interior ideals and weakly almost interior ideals of semigroups and investigated their properties. Almost all ideals of semigroups have attracted much attention from researchers. Iampan et al. [7], Chinram and Nakkhasen [8], Gaketem [9], and Gaketem and Chinram [10] proposed the ideas of almost subsemigroups, almost bi-quasi-interior ideals; almost bi-interior ideals and almost bi-quasi ideals of semigroups, respectively.

Additionally, many researchers studied different types of almost ideal fuzzification [5], [7]–[12]. Molodtsov [13] proposed the idea of a soft set as a function from the parameter set E to the power set of U to model uncertainty. Since then, soft sets have attracted the interest of researchers in many fields. In [14]–[22], soft set operations, the basic idea of the theory, were studied. Çağman and Enginoğlu [23] modified the definition of soft set and, thus, soft set operations. Moreover, several soft algebraic systems have been studied using the soft intersection groups introduced by Çağman et al. [24]. The idea of utilization of soft sets in semigroup theory was by Sezer et al. [25], [26]. In [25], [26], soft intersection subsemigroups left (right/sided ideals), (generalized) bi-ideals, interior ideals, and quasi-ideals of semigroups were studied. Soft sets were also studied as a wide range of algebraic structures in [27]–[34]. Recently, Rao [35–38] brought a few new forms of ideals of semigroups, which include bi-interior ideal, bi-quasi ideal, quasi-interior ideal, bi-quasi interior ideals, weak ideals, and Baupradist et al. [39] defined essential ideals of semigroups.

In this study, we add the perception of soft intersection almost bi-ideal, a generalization of the nonnull soft intersection bi-ideal of semigroups defined in [25]. We obtain that the collection of soft intersections is almost bi-ideal of a semigroup and constructs a semigroup under the binary operation of soft union operation; but not the soft intersection operation. Furthermore, we prove the relationship among almost bi-ideal and soft intersection almost bi-ideal of a semigroup corresponding with minimality, primeness, semiprimeness, and strong primeness by observing that if a nonempty set A is almost quasi-ideal, then its soft characteristic function is soft intersection almost quasi-ideal, and vice versa.

2 | Preliminaries

This section reviews several fundamental notions related to semigroups and soft sets.

Definition 1 ([13], [23]). Let U be the universal set, E be the parameter set, $P(U)$ be the power set of U and $K \subseteq E$. A soft set f_K over U is a set-valued function such that $f_K: E \to P(U)$ such that for all $x \notin K$, $f_K(x) = \emptyset$. A soft set over U can be represented by the set of ordered pairs

$$f_K = \{(x, f_K(x)): x \in E, f_K(x) \in P(U)\}.$$

Throughout this paper, the set of all the soft sets over U is designated by $S_E(U)$.

Definition 2 ([23]). Let $f_A \in S_E(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then f_A is called a null soft set and denoted by $\emptyset_E$. If $f_A(x) = U$ for all $x \in E$, then f_A is called absolute soft set and denoted by U_E.

Definition 3 ([23]). Let f_A, $f_B \in S_E(U)$. If $f_A(x) \subseteq f_B(x)$ for all $x \in E$, then f_A is a soft subset of f_B and denoted by $f_A \tilde{\subseteq} f_B$. If $f_A(x) = f_B(x)$ for all $x \in E$, then f_A is called soft, equal to f_B and denoted by $f_A = f_B$.

Definition 4 ([23]). Let f_A, $f_B \in S_E(U)$. The union of f_A and f_B is the soft set $f_A \tilde{\cup} f_B$, where $(f_A \tilde{\cup} f_B)(x) = f_A(x) \cup f_B(x)$, for all $x \in E$. The intersection of f_A and f_B is the soft set $f_A \tilde{\cap} f_B$, where $(f_A \tilde{\cap} f_B)(x) = f_A(x) \cap f_B(x)$, for all $x \in E$.

Definition 5 ([18]). For a soft set f_A, the support of f_A is defined by $supp(f_A)=\{x \in A : f_A(x) \neq \emptyset\}$.

Thus, a null soft set is indeed a soft set with an empty support, and we say that a soft set f_A is nonnull if $supp(f_A) \neq \emptyset$.

Note: If $f_A \tilde{\subseteq} f_B$, then $supp(f_A) \subseteq supp(f_B)$ [40].

A semigroup S is a nonempty set with an associative binary operation, and throughout this paper, S stands for a semigroup, and all the soft sets are the elements of $S_S(U)$ unless otherwise specified. A nonempty subset A of S is called a subsemigroup of S if $AA \subseteq A$; and is called a bi-ideal of S if $ASA \subseteq A$; A nonempty subset A of S is called an almost bi-ideal of S if $AsA \cap A \neq \emptyset$ for all $s \in S$. An almost bi-ideal A of S is called a minimal almost bi-ideal of S if for any almost bi-ideal B of S if whenever $B \subseteq A$, then $A = B$. An almost bi-ideal P of S is called a prime almost bi-ideal if for any almost bi-ideals A and B of S such that $AB \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$. A bi-ideal P of S is called a semiprime almost bi-ideal if any almost bi- ideal A of S such that $AA \subseteq P$ implies that $A \subseteq P$. An almost bi-ideal P of S is a strongly prime almost bi-ideal if for any almost bi-ideals A and B of S such that $AB \cap BA \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

Definition 6 ([25]). Let f_S and g_S be soft sets over the common universe U. Then, the soft intersection product $f_S \circ g_S$ is defined by

$$(f_S \circ g_S)(x) = \begin{cases} \bigcup\limits_{x=yz} \{f_S(y) \cap g_S(z)\}, & \text{if there exists } y, z \in S \text{ such that } x = yz, \\ \emptyset, & \text{otherwise.} \end{cases}$$

Theorem 1. Let $f_S, g_S, h_S \in S_S(U)$. Then

I. $(f_S \circ g_S) \circ h_S = f_S \circ (g_S \circ h_S)$.

II. $f_S \circ g_S \neq g_S \circ f_S$, generally.

III. $f_S \circ (g_S \tilde{\cup} h_S) = (f_S \circ g_S) \tilde{\cup} (f_S \circ h_S)$ and $(f_S \tilde{\cup} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cup} (g_S \circ h_S)$.

IV. $f_S \circ (g_S \tilde{\cap} h_S) = (f_S \circ g_S) \tilde{\cap} (f_S \circ h_S)$ and $(f_S \tilde{\cap} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cap} (g_S \circ h_S)$.

V. If $f_S \tilde{\subseteq} g_S$, then $f_S \circ h_S \tilde{\subseteq} g_S \circ h_S$ and $h_S \circ f_S \tilde{\subseteq} h_S \circ g_S$.

VI. If $t_S, k_S \in S_S(U)$ such that $t_S \tilde{\subseteq} f_S$ and $k_S \tilde{\subseteq} g_S$, then $t_S \circ k_S \tilde{\subseteq} f_S \circ g_S$ [25].

Lemma 1. Let f_S and g_S be soft sets over U. Then, $f_S \circ g_S = \emptyset_S \Leftrightarrow f_S = \emptyset_S$ or $g_S = \emptyset_S$ [41].

Definition 7. Let A be a subset of S. We denote by S_A the soft characteristic function of A and define as

$$S_A(x) = \begin{cases} U, & \text{if } x \in A, \\ \emptyset, & \text{if } x \in S \backslash A. \end{cases}$$

The soft characteristic function of A is a soft set over U, that is, $S_A: S \longrightarrow P(U)$ [25].

Corollary 1. $supp(S_A) = A$ [40].

Theorem 2. Let X and Y be nonempty subsets of S. Then, the following properties hold [25,40]:

I. $X \subseteq$ if and only if $S_X \tilde{\subseteq} S_Y$.

II. $S_X \tilde{\cap} S_Y = S_{X \cap Y}$ and $S_X \tilde{\cup} S_Y = S_{X \cup Y}$, $S_X \circ S_Y = S_{XY}$.

Definition 8 ([41]). Let x be an element in S. We denote by S_x the soft characteristic function of x and define as

$$S_x(y) = \begin{cases} U, & \text{if } y = x, \\ \emptyset, & \text{if } y \neq x. \end{cases}$$

The soft characteristic function of x is a soft set over U, that is, $S_x: S \longrightarrow P(U)$.

Corollary 2. Let $x \in S$, f_S and S_x be soft sets over U. Then, $f_S \circ S_X \circ f_S = \emptyset_S \Leftrightarrow f_S = \emptyset_S$.

Proof: By *Lemma 1*, $f_S \circ S_X \circ f_S = \emptyset_S \Leftrightarrow f_S = \emptyset_S$ or $S_x = \emptyset_S$. By *Definition 8*, $S_x \neq \emptyset_S$; hence, the rest of the proof is obvious.

Definition 9 ([25]. A soft set f_S over U is called a soft intersection bi-ideal of S over U if $f_S(xy) \supseteq f_S(x) \cap f_S(y)$ and $f_S(xyz) \supseteq f_S(x) \cap f_S(z)$ for all x, y, z ∈ S.

It is easy to see that if $f_S(x) = U$ for all $x \in S$, then f_S is a soft intersection bi-ideal of S. We denote such a kind of soft intersection bi-ideal by $\mathbb{S}$. It is obvious that $\mathbb{S} = S_S$, that is, $\mathbb{S}(x) = U$ for all x ∈ S [25].

For the sake of brevity, soft intersection bi-ideal is abbreviated by SI-B-ideal in what follows.

Theorem 3. Let f_S be a soft set over U. Then, f_S is an SI-B-ideal of S over U if and only if $f_S \circ f_S \tilde{\subseteq} f_S$ and $f_S \circ \mathbb{S} \circ f_S \tilde{\subseteq} f_S$ [25].

Definition 10 ([40]. A soft set f_s is called a soft intersection almost subsemigroup of S if $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ for all x ∈ S.

We refer to [42] for the possible implications of network analysis and graph applications with regard to soft sets, which are defined by the divisibility of determinants.

3 | Soft Intersection Almost Bi-ideals

Definition 11. A soft set f_s is called an almost soft intersection bi-ideal of S if

$$(f_S \circ S_X \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$$

for all x ∈ S. For the sake of ease, soft intersection almost bi-ideal is abbreviated by SI-almost B-ideal in what follows.

Example 1. Let $S = \{n, r\}$ be the semigroup with the following Cayley Table:

	n	r
n	n	r
r	r	n

Let f_S be soft set over $U = \{\left[\begin{smallmatrix} 0 & t \\ 0 & t \end{smallmatrix}\right] \mid t \in \mathbb{Z}_3\}$ as follows:

$f_S = \{(n, \{\left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 1 \\ 0 & 1 \end{smallmatrix}\right]\}), (r, \{\left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 2 \\ 0 & 2 \end{smallmatrix}\right]\})\}$.

Here, f_S is an SI-almost B-ideal, that is, for all x ∈ S, $(f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_s$.

Let's start with S_n:

$[(f_S \circ S_n \circ f_S) \tilde{\cap} f_S](n) = (f_S \circ S_n \circ f_S)(n) \cap f_S(n) = [((f_S \circ S_n)(n) \cap f_S(n)) \cup ((f_S \circ S_n)(r) \cap f_S(r))] \cap f_S(n) = \{[(f_S(n) \cap S_n(n)) \cup (f_S(r) \cap S_n(r))] \cap f_S(n)\} \cup \{[(f_S(r) \cap S_n(n)) \cup (f_S(n) \cap S_n(r)] \cap f_S(r)\} \cap f_S(n) = [(f_S(n) \cap f_S(n)) \cup (f_S(r) \cap f_S(r))] \cap f_S(n) = [f_S(n) \cup f_S(r)] \cap f_S(n) = f_S(n) = \{\left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 1 \\ 0 & 1 \end{smallmatrix}\right]\}$.

$[(f_S \circ S_n \circ f_S) \tilde{\cap} f_S](r) = (f_S \circ S_n \circ f_S)(r) \cap f_S(r) = [((f_S \circ S_n)(r) \cap f_S(n)) \cup ((f_S \circ S_n)(n) \cap f_S(r))] \cap f_S(r) = \{[(f_S(r) \cap S_n(n)) \cup (f_S(n) \cap S_n(r))] \cap f_S(n)\} \cup \{[(f_S(n) \cap S_n(n)) \cup (f_S(r) \cap S_n(r)] \cap f_S(r)\} \cap f_S(r) = [(f_S(r) \cap f_S(n)) \cup (f_S(n) \cap f_S(r))] \cap f_S(r) = [f_S(n) \cap f_S(r)] \cap f_S(r) = f_S(n) \cap f_S(r) = \{\left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right]\}$.

Thus, $(f_S \circ S_n \circ f_S) \tilde{\cap} f_S = \{(n, \{\left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0 & 1 \\ 0 & 1 \end{smallmatrix}\right]\}), (r, \{\left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right]\})\} \neq \emptyset_s$.

Let's continue with S_r:

$[(f_S \circ S_r \circ f_S) \tilde{\cap} f_S](n) = (f_S \circ S_r \circ f_S)(n) \cap f_S(n) = [((f_S \circ S_r)(n) \cap f_S(n)) \cup ((f_S \circ S_r)(r) \cap f_S(r))] \cap f_S(n) = \{ [(f_S(n) \cap S_r(n)) \cup (f_S(r) \cap S_r(r))] \cap f_S(n)\} \cup \{ [(f_S(r) \cap S_r(n)) \cup (f_S(n) \cap S_r(r)] \cap f_S(r)\} \cap f_S(n) = [(f_S(r) \cap f_S(n)) \cup (f_S(n) \cap f_S(r))] \cap f_S(n) = [f_S(n) \cap f_S(r)] \cap f_S(n) = f_S(n) \cap f_S(r) = \{ [\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}].$

$[(f_S \circ S_r \circ f_S) \tilde{\cap} f_S](r) = (f_S \circ S_r \circ f_S)(r) \cap f_S(r) = [((f_S \circ S_r)(r) \cap f_S(n)) \cup ((f_S \circ S_r)(n) \cap f_S(r))] \cap f_S(r) = \{ [(f_S(r) \cap S_r(n)) \cup (f_S(n) \cap S_r(r))] \cap f_S(n)\} \cup \{ [(f_S(n) \cap S_r(n)) \cup (f_S(r) \cap S_r(r)] \cap f_S(r)\} \cap f_S(r) = [(f_S(n) \cap f_S(n)) \cup (f_S(r) \cap f_S(r))] \cap f_S(r) = [f_S(n) \cup f_S(r)] \cap f_S(r) = f_S(r) = \{ [\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}], [\begin{smallmatrix} 0 & 2 \\ 0 & 2 \end{smallmatrix}]\}.$

Thus, $(f_S \circ S_n \circ f_S) \tilde{\cap} f_S = \{(n, \{ [\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}] \}), (r, \{[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}], [\begin{smallmatrix} 0 & 2 \\ 0 & 2 \end{smallmatrix}]\})\} \neq \emptyset_S$.

Therefore, $(f_S \circ S_X \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x \in S$. Thus, f_S is an SI-almost B-ideal.

Example 2. Let $S = \{k, m, n\}$ be the semigroup with the following Cayley Table:

	k	M	n
k	m	N	m
m	n	m	n
n	m	n	m

Let g_S be soft set over $U = \mathbb{N}$ as follows: $g_S = \{ (k, \{1,2\}), (m, \{3,4\}), (n, \emptyset)\}$.

Here, g_S is not an SI-almost B-ideal. In deed; $[(g_S \circ S_k \circ g_S) \tilde{\cap} g_S](k) = \emptyset$. Moreover

$[(g_S \circ S_k \circ g_S) \tilde{\cap} g_S](m) = (g_S \circ S_k \circ g_S)(m) \cap g_S(m) = \{ [(g_S \circ S_k)(k) \cap g_S(k)] \cup [(g_S \circ S_k)(k) \cap g_S(n)] \cup [(g_S \circ S_k)(m) \cap g_S(m)] \cup [(g_S \circ S_k)(n) \cap g_S(k)] \cup [(g_S \circ S_k)(n) \cap g_S(n)] \} \cap g_S(m) = \{ \emptyset \cup \emptyset \cup \{ [g_S(k) \cap S_k(k)] \cup [g_S(k) \cap S_k(n)] \cup [g_S(m) \cap S_k(m)] \cup [g_S(n) \cap S_k(k)] \cup [g_S(n) \cap S_k(n)] \cap g_S(m)\} \cup \{[g_S(k) \cap S_k(m)] \cup [g_S(m) \cap S_k(k)] \cup [g_S(m) \cap S_k(n)] \cup [g_S(n) \cap S_k(m)] \cap g_S(k)\} \cup \{[g_S(k) \cap S_k(m)] \cup [g_S(m) \cap S_k(k)] \cup [g_S(m) \cap S_k(n)] \cup [g_S(n) \cap S_k(m)] \cap g_S(n)] \cap g_S(m)\} = \{ \emptyset \cup \emptyset \cup \{[g_S(k) \cup g_S(n)] \cap g_S(m)] \cup [g_S(m) \cap g_S(k)] \cup [g_S(m) \cap g_S(n)] \} \cap g_S(m)\} = [\emptyset \cup \emptyset \cup \emptyset \cup \emptyset \cup \emptyset] \cap g_S(m) = \emptyset,$

$[(g_S \circ S_k \circ g_S) \tilde{\cap} g_S](n) = (g_S \circ S_k \circ g_S)(n) \cap g_S(n) = (g_S \circ S_k \circ g_S)(n) \cap \emptyset = \emptyset.$

Hence, $(g_S \circ S_X \circ g_S) \tilde{\cap} g_S = \emptyset_S$ for all $x \in S$. Thus, g_S is not an SI-almost B-ideal.

Proposition 1. If f_S is an SI-B-ideal such that $f_S \neq \emptyset_S$, then f_S is an SI-almost B-ideal.

Proof: Let $f_S \neq \emptyset_S$ be an SI-B-ideal. Then, $f_S \circ f_S \tilde{\subseteq} f_S$ ve $f_S \circ \mathbb{S} \circ f_S \tilde{\subseteq} f_S$. Since $f_S \neq \emptyset_S$, by *Corollary 2*, it follows that $f_S \circ S_X \circ f_S \neq \emptyset_S$. We need to show that $(f_S \circ S_X \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ for all $x \in S$.

Since $f_S \circ S_X \circ f_S \subseteq f_S \circ \mathbb{S} \circ f_S \subseteq f_s$ it follows that $f_S \circ S_X \circ f_S \subseteq f_S$. Thus $(f_S \circ S_X \circ f_S) \tilde{\cap} f_S = f_S \circ S_X \circ f_S \neq \emptyset_S$

Therefore, f_S is an SI- almost B-ideal.

It is obvious that $\emptyset_S$ is an SI-B-ideal as $\emptyset_S \circ \emptyset_S \tilde{\subseteq} \emptyset_S$ ve $\emptyset_S \circ \mathbb{S} \circ \emptyset_S \tilde{\subseteq} \emptyset_S$; but it is not SI-almost B-ideal since $(\emptyset_S \circ S_x \circ \emptyset_S) \tilde{\cap} \emptyset_S = \emptyset_S \tilde{\cap} \emptyset_S = \emptyset_S$.

Here, note that if f_S is an SI-almost B-ideal, then f_S needs not to be an SI-B-ideal, as shown in the following example:

Example 3. In *Example 2*, it is shown that f_S is SI-almost B-ideal; however f_S is not SI-B-ideal. Indeed

$(f_S \circ \mathbb{S} \circ f_S)(n) = [(f_S \circ \mathbb{S})(n) \cap f_S(n)] \cup [(f_S \circ \mathbb{S})(r) \cap f_S(r)] = [[(f_S(n) \cap \mathbb{S}(n)) \cup (f_S(r) \cap \mathbb{S}(r)] \cap f_S(n)] \cup [[(f_S(r) \cap \mathbb{S}(n)) \cup (f_S(n) \cap \mathbb{S}(r))] \cap f_S(r)] = [(f_S(n) \cup f_S(r)) \cap f_S(n)] \cup [(f_S(r) \cup f_S(n)) \cap f_S(r)] = f_S(n) \cup f_S(r) \not\subseteq f_S(n)$. Thus, f_S is not an SI-B-ideal.

Proposition 2. Let f_S be an idempotent SI-almost B-ideal. Then, f_S is an SI-almost subsemigroup.

Proof: Assume that f_S is an idempotent SI-almost B-ideal. Then, $f_S \circ f_S = f_S$ and $(f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ for all $x \in S$. We need to show that $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ for all $x \in S$. Since, $\emptyset_S \neq (f_S \circ S_x \circ f_S) \tilde{\cap} f_S = [(f_S \circ S_x \circ f_S) \tilde{\cap} f_S] \tilde{\cap} f_S = [(f_S \circ S_x \circ f_S) \tilde{\cap} (f_S \circ f_S)] \tilde{\cap} f_S \tilde{\subseteq} (f_S \circ f_S) \tilde{\cap} f_S$, hence f_S is an SI-almost subsemigroup.

Theorem 4. Let $f_S \tilde{\subseteq} g_S$. If f_S is an SI-almost B-ideal, then g_S is an SI-almost B-ideal.

Proof: Assume that f_S is an SI-almost B-ideal. Hence, $(f_S \circ S_X \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$ for all $x \in S$. We need to show that $(g_S \circ S_X \circ g_S) \tilde{\cap} g_S \neq \emptyset_S$.

In fact, $(f_S \circ S_X \circ f_S) \tilde{\cap} f_S \tilde{\subseteq} (g_S \circ S_X \circ g_S) \tilde{\cap} g_S$. Since $(f_S \circ S_X \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$. it is obvious that $(g_S \circ S_X \circ g_S) \tilde{\cap} g_S \neq \emptyset_S$. Thus g_S is an SI-almost B-ideal.

Theorem 5. Let f_S and g_S be SI-almost B-ideals. Then, $f_S \tilde{\cup} g_S$ is an SI-almost B-ideal.

Proof: Let f_S and g_S SI-almost B-ideals. Since, $f_S \tilde{\subseteq} f_S \tilde{\cup} g_S$, $f_S \tilde{\cup} g_S$ is an SI-almost B-ideal by *Theorem 4*.

Corollary 3. Let f_S and g_S be soft sets over U. Then, we have the following:

I. If f_S or g_S be SI-almost B-ideals. then $f_S \tilde{\cup} g_S$ is an SI-almost B-ideal.

II. The finite union of SI-almost B-ideals is an SI-almost B-ideal.

Here, note that if f_S and g_S are SI-almost B-ideals, then $f_S \tilde{\cap} g_S$ needs not to be an SI-almost B-ideal.

Example 4. Let $S = \{n, r\}$ be the semigroup in *Example 3*, and h_S and t_S be soft sets over $U = \{\begin{bmatrix} 0 & c \\ 0 & c \end{bmatrix} \mid c \in \mathbb{Z}_3\}$ as follows:

$h_S = \{(n, \{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\}), (r, \{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\})\}$,
$t_S = \{(n, \{\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}\}), (r, \{\begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}\})\}$.

One can easily see that h_S and t_S are SI-almost B-ideals. However, $h_S \tilde{\cap} t_S = \{(n, \emptyset), (r, \emptyset)\}$. Therefore, $h_S \tilde{\cap} t_S$ is not an SI-almost B-ideal.

Now, we give the relationship between almost B-ideal and SI-almost B-ideal of S. But first, we remind the following lemma to use in *Theorem 6*.

Lemma 2. Let $x \in S$ and Y be a nonempty subset of S. Then, $S_x \circ S_Y = S_{xY}$ [42].

Theorem 6. Let A be a nonempty subset of S. Then, A is an almost B-ideal if and only if S_A, the soft characteristic function of A, is an SI-almost B-ideal.

Proof: Assume that $\emptyset \neq A$ is an almost B-ideal. Then, $AxA \cap A \neq \emptyset$ for all $x \in S$, and so there exits $k \in S$ such that $k \in AxA \cap A$. Since, $((S_A \circ S_x \circ S_A) \tilde{\cap} S_A)(k) = (S_{AxA} \tilde{\cap} S_A)(k) = (S_{AxA \cap A})(k) = U \neq \emptyset$.

It follows that $(S_A \circ S_x \circ S_A) \tilde{\cap} S_A \neq \emptyset_S$. Thus, S_A is an SI-almost B-ideal.

Conversely, assume that S_A is an SI-almost B-ideal. Hence, we have $(S_A \circ S_x \circ S_A) \tilde{\cap} S_A \neq \emptyset_s$, for all $x \in S$. In order to show that A is an almost B-ideal, we should prove that $A \neq \emptyset$ and $AxA \cap A \neq \emptyset$, for all $x \in S$. $A \neq \emptyset$ is obvious from the assumption. Now,

$\emptyset_s \neq (S_A \circ S_x \circ S_A) \tilde{\cap} S_A \Longrightarrow$ there exists $k \in S$; $(S_A \circ S_x \circ S_A) \tilde{\cap} S_A)(k) \neq \emptyset$,
$\Longrightarrow$ there exists $k \in S$; $(S_{AxA} \tilde{\cap} S_A)(k) \neq \emptyset$,
$\Longrightarrow$ there exists $k \in S$; $(S_{AxA \cap A})(k) \neq \emptyset$,
$\Longrightarrow$ there exists $k \in S$; $(S_{AxA \cap A})(k) = U$,
$\Longrightarrow$ $k \in AxA \cap A$.

Hence, $AxA \cap A \neq \emptyset$ and so, A is an almost B-ideal.

Lemma 3. Let f_S be a soft set over U. Then, $f_S \tilde{\subseteq} S_{supp(f_S)}$ [41].

Theorem 7. If f_s is an SI-almost B-ideal, then $supp(f_S)$ is an almost B-ideal.

Proof: Assume that f_s is an SI-almost B-ideal. Thus, $(f_S \circ S_X \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$. In order to show that $supp(f_S)$ is an almost B-ideal, by *Theorem 6*, it is enough to show that $S_{supp(f_S)}$ is an SI-almost B-ideal. Since, $(f_S \circ S_X \circ f_S) \tilde{\cap} f_S \tilde{\subseteq} (S_{supp(f_S)} \circ S_X \circ S_{supp(f_S)}) \tilde{\cap} S_{supp(f_S)}$ and $(f_S \circ S_X \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, it implies that $(S_{supp(f_S)} \circ S_X \circ S_{supp(f_S)}) \tilde{\cap} S_{supp(f_S)} \neq \emptyset_S$. Consequently, $S_{supp(f_S)}$ is an SI-almost B-ideal and by *Theorem 6*, $supp(f_S)$ is an almost B-ideal.

Here, note that the converse of *Theorem 7* is not true in general, as shown in the following example.

Example 5. We know that g_S is not an SI-almost B-ideal in *Example 2.* and it is evident that $supp(g_S) = \{k, m\}$. Since

$$[supp(g_S)\ \{k\}\ supp(g_S)] \cap supp(g_S) = \{k, m\}\ k\ \{k, m\} \cap \{k, m\} = \{m, n\}\{k, m\} \cap \{k, m\} = \{m, n\} \cap \{k, m\} = \{m\} \neq \emptyset,$$
$$[supp(g_S)\ \{m\}\ supp(g_S)] \cap supp(g_S) = \{k, m\}\ m\ \{k, m\} \cap \{k, m\} = \{m, n\}\{k, m\} \cap \{k, m\} = \{m, n\} \cap \{k, m\} = \{m\} \neq \emptyset,$$
$$[supp(g_S)\ \{n\} supp(g_S)] \cap supp(g_S) = \{k, m\}\ n\ \{k, m\} \cap \{k, m\} = \{m, n\}\{k, m\} \cap \{k, m\} = \{m, n\} \cap \{k, m\} = \{m\} \neq \emptyset.$$

It is seen that $[supp(g_S)\, x supp(g_S)] \cap supp(g_S) \neq \emptyset$, for all $x \in S$. That is to say, $supp(g_S)$ is an almost B-ideal, although g_S is not an SI-almost B-ideal.

Definition 12. An SI-almost B-ideal f_S is called minimal if any SI-almost B-ideal h_S if whenever $h_S \tilde{\subseteq} f_S$, then $supp(h_S) = supp(f_S)$.

Theorem 8. Let A be a nonempty subset of S. Then, A is a minimal almost B-ideal if and only if S_A , the soft characteristic function of A, is a minimal SI- almost B-ideal.

Proof: Assume that A is a minimal, almost B-ideal. Thus, A is an almost B-ideal, and so S_A is an SI-almost B-ideal by *Theorem 6*. Let f_S be an SI-almost B-ideal such that $f_S \subseteq S_A$. By *Theorem 6*, $supp(f_S)$ is an almost B-ideal, and by note and *Corollary 1*, $supp(f_S) \subseteq supp(S_A) = A$.

Since A is a minimal, almost B-ideal, $supp(f_S) = supp(S_A) = A$. Thus, S_A is a minimal SI-almost B-ideal by *Definition 12*.

Conversely, let S_A be a minimal SI-almost B-ideal. Thus, S_A is an SI-almost B-ideal, and A is an almost B-ideal by *Theorem 6*. Let B be an almost B-ideal such that $B \subseteq A$. By *Theorem 6*, S_B is an SI-almost B-ideal, and by *Theorem 3*, $S_B \tilde{\subseteq} S_A$. Since S_A is a minimal SI-almost B-ideal, $B = supp(S_B) = supp(S_A) = A$ by *Corollary 1*. Thus, A is a minimal, almost B-ideal.

Definition 13. Let f_S, g_S, and h_S be any SI-almost B-ideals. If $h_S \circ g_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-prime almost B-ideal.

Definition 14. Let f_S and h_S be any SI- almost B-ideals. If $h_S \circ h_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$, then f_S is called an SI-semiprime almost B-ideal.

Definition 15. Let f_S, g_S, and h_S be any SI- almost B-ideals. If $(h_S \circ g_S) \tilde{\cap} (g_S \circ h_S) \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-strongly prime almost B-ideal.

Obviously, every SI-strongly prime almost B-ideal is an SI-prime almost B-ideal, and every SI-prime almost B-ideal is a soft semiprime almost B-ideal.

Theorem 9. If S_P, the soft characteristic function of P, is an SI-prime almost B-ideal, then P is a prime almost B-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-prime almost B-ideal. Thus, S_P is an SI-almost B-ideal, and thus, P is an almost B-ideal by *Theorem 6*. Let A and B be almost B-ideal such that $AB \subseteq P$. Thus, by *Theorem 6*, S_A and S_B are SI-almost B-ideal and $S_A \circ S_B = S_{AB} \tilde{\subseteq} S_P$. Since S_P is an SI-prime almost B-ideal and $S_A \circ S_B \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$ or $S_B \tilde{\subseteq} S_P$. Therefore, by *Theorem 3*, $A \subseteq P$ or $B \subseteq P$. Consequently, P is a prime, almost B-ideal.

Theorem 10. If S_P, the soft characteristic function of P, is an SI-semiprime almost B-ideal, then P is a semiprime almost bi-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-semiprime almost B-ideal. Thus, S_P is an SI-almost B-ideal, and thus, P is an almost B-ideal by *Theorem 6*. Let A be an almost B-ideal such that $AA \subseteq P$. Thus, by *Theorem 6*, S_A is an SI-almost B-ideal and $S_A \circ S_A = S_{AA} \tilde{\subseteq} S_P$. Since S_P is an SI-semiprime almost B-ideal and $S_A \circ S_A \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$. Therefore, by *Theorem 3*, $A \subseteq P$. Consequently, P is a semiprime almost B-ideal.

Theorem 11. If S_P, the soft characteristic function of P, is an SI-strongly prime almost B-ideal, then P is a strongly prime almost B-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-stronlgy prime almost B-ideal. Thus, S_P is an SI-almost B-ideal, and thus, P is an almost B-ideal by *Theorem 6*. Let A and B be almost B-ideal such that $AB \cap BA \subseteq P$. Thus, *Theorem 3*, S_A and S_B are SI-almost B-ideal and $(S_A \circ S_B) \tilde{\cap} (S_B \circ S_A) = S_{AB} \tilde{\cap} S_{BA} = S_{AB \cap BA} \tilde{\subseteq} S_P$.

Since S_P is an SI-strongly prime, almost B-ideal and $(S_A \circ S_B) \tilde{\cap} (S_B \circ S_A) \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$ or $S_B \tilde{\subseteq} S_P$. Thus, by *Theorem 3*, $A \subseteq P$ or $B \subseteq P$. Therefore, P is a strongly prime, almost B-ideal.

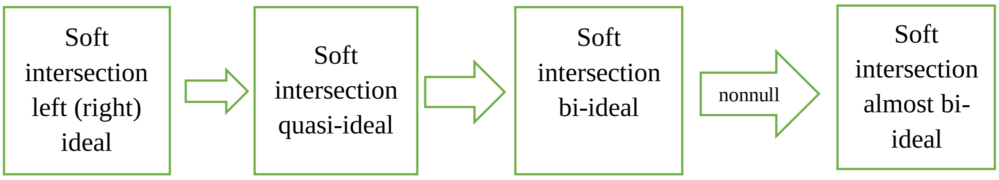

Fig. 1. Relations of the Certain Soft Intersection Ideals.

4 | Conclusion

In this study, we introduced the concept of soft intersection, which is almost bi-ideal, and studied its basic properties. We illustrated that every soft intersection bi-ideal of a semigroup is a soft intersection almost bi-ideal of S; nevertheless, the converse does not hold. We also obtained the relation among soft intersection almost bi-ideal and almost bi-ideal according to minimality, primeness, semiprimeness, and strong primeness by observing that if a nonempty set A is almost quasi-ideal, then its soft characteristic function is soft intersection almost quasi-ideal and vice versa. In the following studies, many almost ideal soft intersections may be examined.

Author Contribution

A. S. research design, methodology, and validation. B. O. conceptualization, reviewing, and editing. The authors have read and agreed to the published version of the manuscript

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All data supporting the reported findings in this research paper are provided within the manuscript.

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